Long-Term Fluctuations in the Earth's Rotation: 700 BC to AD 1990

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Long-term fluctuations in the Earth's rotation: 700 BC to AD 1990

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Contents

Records of solar and lunar eclipses in the period 700 BC to AD 1600, originating from the ancient and medieval civilizations of Babylon, China, Europe and the Arab world, are amassed and critically appraised for their usefulness in answering questions about the long-term variability of the Earth's rate of rotation. Results from previous analyses of lunar occultations in the period AD 1600–1955.5, and from high-precision data in AD 1955.5–1990, are included in the dataset considered in this paper.

Using the change in the length of the mean solar day (l.o.d.) in units of milliseconds per century (ms cy^{-1}) as the measure of acceleration in the rate of rotation, it is found that the l.o.d. has increased by $(+1.70 \pm 0.05)$ ms cy^{-1} ($\equiv (-4.5 \pm 0.1) \times 10^{-22}$ rad s^{-2}) on average over the past 2700 years. Yet an increase of $+2.3 \pm 0.1$ ms cy^{-1} ($\equiv (-6.1 \pm 0.4) \times 10^{-22}$ rad s^{-2}) is expected from the tidal braking of the Earth's spin, assuming a value of $-26.0''\,\mathrm{cy}^{-2}$ for the tidal acceleration of the Moon.

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There is thus an average accelerative component in the Earth's rotation which acts to decrease the l.o.d. by (-0.6 ± 0.1) ms cy^{-1} ($\equiv (+1.6\pm0.4)\times10^{-22}$ rad s^{-2}). Moreover, it is shown that besides this accelerative component, there is a fluctuation in the l.o.d. with a semi-amplitude of $\sim$ 4 ms and a period of $\sim$ 1500 yr.

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1. Introduction

> 'For in and near these places (Baghdad, Aleppo and Alexandria) were made all the observations whereby the middle motions of the Sun and Moon are limited. And I could then pronounce in what proportion the Moon's motion does accelerate: which that it does, I think I can demonstrate, and shall (God willing) demonstrate to the publick.'

So wrote Edmond Halley three hundred years ago (1695). Though he never appears to have realized his objective – at least in his published papers – he seems to have been the first to appreciate the importance of ancient and medieval observations in determining the acceleration of the Moon.

Only in the last ten years or so has the acceleration of the Moon been measured accurately from lunar laser ranging and the observed perturbations on the orbits of artificial satellites caused by lunar tides. By the principle of conservation of angular momentum in the Earth–Moon system, the angular momentum gained by the Moon's orbit is lost by the spinning Earth. The mechanism through which the Earth spins down on geological timescales is lunar and solar tidal dissipation in the oceans and solid body of the Earth. Besides this dominant evolutionary mechanism affecting the Earth's rotation, changes on timescales from days to decades have been measured from astronomical observations. The redistribution of angular momentum within the Earth through the mechanism of core–mantle coupling is the most likely cause of the decadal variations. Changes on timescales from a few days to years are due to the exchange of angular momentum between the atmosphere and shell/mantle.

High-precision data generated by modern astronomical techniques since the fifties enable very detailed studies to be made of the short-term variations in the Earth's rotation over this recent period. Data of considerably lower precision, which have been available since the start of telescopic observations at around AD 1620, permit the decadal variations to be studied in some detail. However, on timescales of centuries to millennia, much less is known about variations in the Earth's rotation because of the imprecision and sparseness of suitable observations.

In a previous work (Stephenson & Morrison 1984) we assembled and analysed astronomical data in the period 700 BC to AD 1980 and showed that, besides the deceleration in the Earth's spin due to tidal dissipation, there was, in addition, a smaller accelerative component over the past 2700 years. We attempted to measure the size of this acceleration and came to the tentative conclusion that it was variable on a timescale of millennia. However, the evidence was not conclusive because of the paucity of suitable data, particularly in the first millennium of the Christian era. Recently, this has been rectified by the provenance of further observations, especially from Babylon and China. These data, combined with a thorough reappraisal of the previously known Babylonian (*ca.* 700–50 BC), Chinese (*ca.* 200 BC to AD 1300), European (*ca.* AD 800–1600) and Arabian (*ca.* AD 800–1200) data, permit us to separate the accelerative component and its variation with time from the underlying deceleration due to tidal dissipation.

These results, as well as increasing our knowledge of the history of the Earth's rotation, set constraints on various geophysical mechanisms which influence the rate of rotation of the Earth, and should act as a spur to further studies.

2. Principles of the method of analysis

The observed period of rotation of the Earth relative to the direction of the Sun is the apparent solar day. Its length varies in an annual cycle, not because of actual variations in spin rate, but because of the geometrical properties of the Earth's motion in an ellipse and the tilt of the equator to the plane of its orbit. When these well-known effects are removed from the observations, a standardized measure of the actual period of rotation is obtained. This period is called the mean solar day and throughout this paper the length of the day (abbreviated to l.o.d.) refers to the mean solar day.

(*a*) *Timescales*

The timescale constructed from the observed l.o.d. is known as universal time (UT). It departs from uniformity by the integral of the changes in the l.o.d. In order to measure the departures from uniformity, a standard of comparison is required, and this standard has to be available over millennia if long-term variations are to be detected.

In principle, the independent variable in the gravitational theories of the motions of the Sun (the reflected motion of the Earth), Moon and planets provides a uniform measure of time. This variable appears explicitly as the argument of time in the fundamental ephemerides of the planetary system, and for this reason was originally known as ephemeris time (ET).

The unit of the ET scale was chosen to be equal to that of Newcomb's solar ephemeris (Newcomb 1895*a*) which was still in general use in 1960 when ET was introduced as the argument of time in the fundamental ephemerides. In Newcomb's ephemeris, the mean motion of the Sun was expressed in the unit of the average length of the second (or, equivalently, the l.o.d.) of the UT scale during the latter half of the 18th and most of the 19th century. This resulted from the fitting of his theory to observations of the position of the Sun made relative to UT during the years 1750–1892 (Newcomb 1895*b*). Thus, the average l.o.d. on the UT scale during the latter half of the 18th and most of the 19th century became the standard length of the day (86 400 s) of the ET scale.

A resonant frequency of the caesium atom was introduced in July 1955 as a new and highly accurate standard of timekeeping, and the International Atomic Time (TAI) scale was generated from this frequency. The number of cycles of this frequency was measured relative to the second of the ET scale by Markowitz *et al.* (1958). Later, the second of the International System of Units (SI) was defined to be that of the TAI scale, and the day of 86 400 s SI became the standard of comparison. Although the unit of time of the TAI scale was initially measured relative to that of the ET scale, no attempt was made to synchronize the two timescales. As a consequence, there is a difference of 32.184 s in epoch between them.

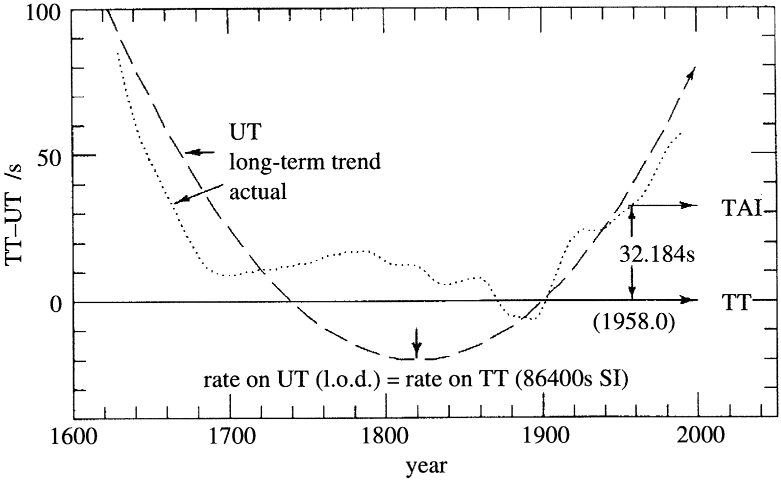

Figure 1. Diagrammatical representation of the relationship between UT, TAI and TT (see text for details).

Recently, ET has been superseded by terrestrial time (TT) as the argument of time of the apparent geocentric ephemerides of the planetary system. In practice, TT, unlike its precursor ET, has by definition precisely the same unit of measure as the TAI scale (86400 s SI), but its origin is offset from it such that

$$\mathrm{TT} = \mathrm{TAI} + 32.184\ \mathrm{s}.$$

The constant, 32.184 s, is included to preserve continuity between the TT scale and its precursor, ET. We shall use TT to denote both the uniform timescale derived from dynamical theory before the introduction of the TAI scale in 1955.5, and also the timescale TT = TAI + 32.184 s, which is directly recoverable from TAI after 1955.5.

The relationship between the various timescales discussed in this section is shown diagrammatically in figure 1. Under the influence of the tidal torque and other long-term forces, UT departs from TT in a nearly parabolic manner. Torques which operate on timescales of days to decades, and possibly longer, cause the actual behaviour of UT to depart from this underlying parabolic trend, as shown in figure 1. The cusp of the parabola should lie somewhere around the middle of the period 1750–1892, in order to fulfil the requirement that the average l.o.d. (i.e. the slope) at that mid epoch should be equal to the standard length of the day on the TT scale (86 400 s SI). For this reason, the cusp is placed near 1820 in figure 1. The fact that UT (actual) is equal to TAI at the epoch 1958.0, and that TT is offset from TAI by 32.184 s, vertically constrains the position of the parabola.

The objective of the present work is to measure the non-uniformity of the UT scale over the past 2700 years (the period covered by comparatively reliable astronomical observations) by reference to the uniform standard provided by TT. The observed times on the UT scale of astronomical events in the solar system are compared with the corresponding times on the TT scale, which are essentially computed from the dynamical equations of motion of the bodies involved in producing the events. Astronomical observations thus yield, directly or indirectly, the difference between TT and UT at the epoch of specific events, and we adopt the usual notation for this difference:

$$\Delta \mathrm{T} = \mathrm{TT} - \mathrm{UT}.$$

The first time derivative of ΔT measures the change in the l.o.d. of the UT scale relative to the reference day of the TT scale, which is 86 400 s SI.

(*b*) *Solar and lunar ephemerides*

Accurate ephemerides of the Sun and Moon stretching back over the past 2700 years are essential to our derivation of long-term changes in the l.o.d. For the Sun's position, we have used Newcomb's analytical ephemeris (Newcomb 1895*a*, with a correction to the mean motion of $+1.05\,''\,\mathrm{cy}^{-1}$). A comparison of the solar positions derived from this ephemeris with those from Bretagnon's (1982) more recent analytical theory produces discrepancies of less than $5\,''$ over the past 2700 years. The accuracy of Newcomb's theory, modified in this way, is certainly adequate for the analysis of eclipse observations made with the unaided eye, which form the basis of our dataset before the 17th century.

The position of the Moon has been taken from the analytical ephemeris designated by $j = 2$ (IAU 1968), with a small parabolic correction to the mean longitude to make it consistent with recent estimates of the tidal acceleration (see below). This ephemeris has been compared with the numerically integrated ephemeris, designated DE102, which was generated at the Jet Propulsion Laboratory (Newhall *et al.* 1983). The difference in longitude is mainly parabolic with time and reaches $\sim 120\,''$ at 700 BC. In latitude, the maximum difference is $\sim 10\,''$. Most of these discrepancies arise from the slightly different value of the lunar tidal acceleration used by us ($-26.0\,''\,\mathrm{cy}^{-2}$) compared with that implicit in DE102 ($-26.2\,''\,\mathrm{cy}^{-2}$). When this parabolic difference is allowed for, the mean discrepancies reduce to $\sim 25\,''$ and $\sim 1\,''$ in longitude and latitude, respectively, and this is small relative to the intrinsic accuracy of the observational material prior to the 17th century.

The value of the tidal acceleration in the Moon's mean longitude, which we designate by $\dot{n}$, is the crucial parameter in the lunar ephemeris from the point of view of the subsequent analysis in this paper. The tidal acceleration dominates the long-term behaviour of the Moon's motion, and it has to be determined empirically. An error in its value translates directly into an error in ΔT, where the lunar ephemeris is involved in reducing observations such as eclipses and occultations. Our analysis covers more than 2500 years and it is therefore necessary to consider the constancy of $\dot{n}$ over this lengthy period.

The acceleration $\dot{n}$ is directly proportional to the rate of dissipation of the Earth's rotational energy by the tides, which occurs largely in the oceans and seas of the Earth. This dissipation of energy would appear to occur mainly in the deep oceans and shallow seas (Lambeck 1980). It is reasonable to suppose that on the millennial timescale there is negligible change in the rate of tidal dissipation in the highly stable deep oceans. There are also grounds for believing that even the shallow sea contribution has remained sensibly contant in the last 2700 years. Estimates of the change in global sea level during this interval do not exceed 1 or 2 m (Fairbridge 1961; Morner 1971), which are small compared with the average depth of water on the sloping continental shelves (100 m). Hence, a change in the area of the shelves over the historical period by more than about 1% seems unlikely. There is thus no good reason for supposing that $\dot{n}$ has varied significantly in the time interval covered by the observations which we have analysed.

The value of $\dot{n}$ in $j = 2$ is $-22.44''\,\mathrm{cy}^{-2}$, which was derived by Spencer Jones (1939) from positions of the Moon and planets, including the transits of Mercury in the period 1677–1927. In a more extensive analysis of the transits of Mercury and lunar occultations in the period 1677–1973, Morrison & Ward (1975) obtained $-26 \pm 2''\,\mathrm{cy}^{-2}$. Recent results from lunar laser ranging (Williams *et al.* 1992) ($-25.9 \pm 0.5''\,\mathrm{cy}^{-2}$) and artificial satellites (see, for example, Christodoulidis *et al.* 1988) ($-25.27 \pm 0.61''\,\mathrm{cy}^{-2}$) support this figure. In this paper we have adopted a value of $-26.0''\,\mathrm{cy}^{-2}$ for the tidal acceleration, and have amended the expression for the mean longitude in $j = 2$ accordingly. An uncertainty in the adopted value of the tidal acceleration of $\pm 0.5''\,\mathrm{cy}^{-2}$ translates directly to an uncertainty of $\pm 0.46\,\mathrm{s\,cy}^{-2}$ in $\Delta\mathrm{T}$. This does not materially affect the conclusions reached in this paper.

For the period from 1830–1990, we have taken the changes in the l.o.d. from a recent analysis of lunar occultation data by Jordi *et al.* (1994). The lunar ephemeris JPL LE200 which was employed in that work has an implicit value of $-23.9''\,\mathrm{cy}^{-2}$ for $\dot{n}$. The correction of $-2.1''\,\mathrm{cy}^{-2}$, to make it consistent with the value we have adopted, requires that a secular rate of $+0.1\,\mathrm{ms\,cy}^{-1}$ be added to the values of the l.o.d. taken from that paper. This is small relative to the decade fluctuations in the l.o.d. in the past 200 years, and has no material significance to the long-term trends discussed in this paper. So, we have taken the results unaltered from Jordi *et al.* (1994).

(*c*) *Tidal friction and the change in the l.o.d.*

From an analysis of the perturbations of near-Earth satellites by lunar and solar tides, and the requirement that angular momentum be conserved in the Earth–Moon system, Christodoulidis *et al.* (1988) found the following empirical relation between the observed tidal acceleration of the Moon $\dot{n}$ and the retardation of the Earth's spin due to lunar and solar tides, $\dot{\Omega}_{\mathrm{tidal}}$:

$$\dot{\Omega}_{\mathrm{tidal}} = +(49 \pm 3) \times 0.004869\dot{n} \times 10^{-22}\ \mathrm{rad\,s}^{-2}.$$

With $\dot{n} = -26.0''\,\mathrm{cy}^{-2}$,

$$\dot{\Omega}_{\mathrm{tidal}} = (-6.20 \pm 0.38) \times 10^{-22}\ \ \mathrm{rad\,s}^{-2}\ .$$

The relationship between the acceleration of the Earth's spin expressed in units of $\mathrm{rad\,s}^{-2}$ and the secular change in the l.o.d., d(l.o.d.)/dt in $\mathrm{ms\,cy}^{-1}$, is (see Stephenson & Morrison 1984, table 2),

$$1\ \mathrm{ms\ cy}^{-1} \equiv -2.67 \times 10^{-22}\ \mathrm{rad\,s}^{-2}.$$

The value of $\dot{\Omega}_{\mathrm{tidal}}$ above is thus equivalent to

$$\mathrm{d(l.o.d.)_{tidal}}/\mathrm{d}t = +2.3 \pm 0.1\ \ \mathrm{ms\ cy}^{-1}\ .$$

In the absence of other torques or changes in the principal moment of inertia, the l.o.d. should increase steadily by $+2.3\,\mathrm{ms\,cy}^{-1}$, and UT should diverge parabolically with time from TT by

$$\mathrm{TT} - \mathrm{UT} = \Delta\mathrm{T}_{\mathrm{tidal}} = ct^2,$$

where c is a constant, and t is time reckoned from the epoch at which the unit of measure of time on the UT scale is equal to that of the TT scale. In other words, it is the epoch at which the l.o.d. is equal to 86 400 seconds SI, and, as we have argued in § 2 *a* above, that epoch is somewhere near the beginning of the 19th century. If t is measured in centuries (cy), then c, measured in s cy^{-2}, is related to the secular change in the l.o.d. measured in ms cy^{-1} (see Stephenson & Morrison 1984, table 2) by

$$\mathrm{d(l.o.d.)}/\mathrm{d}t = 0.0548c.$$

Substituting $+2.3 \pm 0.1$ ms cy^{-1} for d(l.o.d.)/dt, we find

$$\Delta\mathrm{T}_{\mathrm{tidal}} = (+42 \pm 2)t^2 \text{ s},$$

where t is measured in centuries from an epoch circa AD 1800.

(*d*) *Observational requirements for measuring* ΔT

Although calculated changes in the l.o.d. due to tides do not exceed a few tens of ms in the whole of the historical period, the cumulative amount in time as expressed by $42t^2$ s is considerable. At the epoch AD 1000, $\Delta\mathrm{T}_{\mathrm{tidal}}$ amounts to $\sim$ 1 h, and by 500 BC it has increased to $\sim$ 7 h. So, historical observations of astronomical events, from which we can derive ΔT with a resolution better than these amounts at their respective epochs, contain potentially useful information about the Earth's rotation.

Besides considerations of accuracy, an observation must meet the following requirements:

(i) the accurate geographical position of the observer must be known;

(ii) the observation must involve an event in the Solar System, the TT of which is calculable from the dynamical equations of motion of the bodies involved;

(iii) the exact date of the event must be known or capable of being established from other circumstances surrounding the event; and

(iv) The UT, or its equivalent, must be given or calculable from the reported circumstances. The only exceptions to the requirement of UT are large or total solar eclipses, where the narrowness of the eclipse paths on the Earth's surface define the rotational phase of the Earth and, hence, the UT (see § 4 *a* (i)).

Virtually all of the pre-telescopic astronomical observations which satisfy these four conditions involve either eclipses of the Sun or Moon. The main strength of eclipses is that they are so obvious to the unaided eye. Also, apart from large or total solar eclipses – for which timing is not essential – the error in timing an eclipse in the past was minimized by the fact that the measurement was often expressed relative to the local horizon (as the time-interval from sunrise or sunset), or in terms of the altitude of the Sun, Moon or a selected bright star. The instant of sunrise or sunset combined with the geographical position of the observer fixes the position of the Earth on its rotational axis relative to the Sun; i.e. it fixes the UT. So, the clock error only enters the observation through the reported time-interval between sunrise/sunset and the eclipse event. In some instances, this interval is very short and, in a few cases, the eclipse is reported as actually being in progress at sunrise/sunset. The accuracy is then limited by the nature and observability of the eclipse and not the clock error. Similar considerations apply to lunar eclipses. These observational factors, together with those associated with the measurement of altitude, are discussed in the appropriate place in the discussion of the observations in § 3.

Observations other than eclipses have often been analysed in recent years, but the results are unsatisfactory. For example, Newton (1970, pp. 9–29 and 1972*a*) analysed equinox observations made by ancient Greek and medieval Arab astronomers (in addition to eclipses). His results from the equinox data are subject to large uncertainties. Although many records of occultations of planets and stars by the Moon are preserved in ancient and medieval history (largely from China), most are of little value in determining UT. Owing to the brilliance of the Moon, it is difficult for the unaided eye to decide whether or not an occultation is actually taking place unless the occulted object is extremely bright, and often the terminology used in early texts is vague. Additionally, in marked contrast to eclipses, very few occultations in the pre-telescopic period were timed. Fotheringham & Longbottom (1915) analysed several ancient Greek and Roman observations of occultations and close conjunctions of the Moon with stars noted by Ptolemy in his *Almagest* (book VII). However, only two of these involve the occultation of a bright star (α Vir).

In dealing with untimed observations of occultations, Liu Ciyuan (1988*a*, *b*) developed the technique of using a *time window* in which each event was assumed to have occurred at any time on the recorded date when the Moon was above the horizon in a dark sky. Uncertainties in ΔT derived in this way are extremely large (see also Liu & Yau 1990; Hilton *et al.* 1992).

With the start of telescopic observations from AD 1620 onwards, it became possible to reliably witness occultations of stars by the Moon. The combination of the sharpness of the phenomenon and the abundance of recordings in scientific journals, make them the best dataset for measurement of ΔT before the introduction of the TAI scale in July 1955.

(*e*) *Types of observations of eclipses in the pre-telescopic period*

In this work – as in our previous investigations – we have concentrated exclusively on eclipse observations in the pre-telescopic period for the reasons given in § 2 *d*. Fortunately, a wide variety of records of solar and lunar eclipses can be used to measure ΔT.

Potentially viable observations of solar eclipses include the following:

(i) untimed total and annular eclipses;

(ii) untimed large partial eclipses (for which totality or annularity is specifically denied);

(iii) estimates of eclipse magnitude (at greatest phase);

(iv) instances where it is noted that the Sun rose or set while eclipsed; and

(v) measurements of the local time of the various contacts.

To the above list may be added the following types of lunar eclipse observations:

(vi) estimates of the degree of obscuration of the Moon at moonrise or moonset;

(vii) instances where it is simply noted that the Moon rose or set while eclipsed (without any reference to magnitude); and

(viii) measurements of the local time of the various contacts. The magnitude of a lunar eclipse is independent of UT, and therefore estimates of this quantity do not contribute anything to the determination of ΔT.

Before considering the derivation of ΔT from observations in the above categories, the sources of observations are discussed and examples of the various pre-telescopic records are given. Unless otherwise stated, all translations are by the authors.

Table 1. *Numbers of eclipse observations.*

	untimed		timed	
source	solar	lunar	solar	lunar
Babylon	5	33	14	105
China	11	4	27	52
Europe	21	—	3	17
Arab	4	—	18	25

3. Observations

(*a*) *Pre-telescopic records of eclipses*

Pre-telescopic records of eclipses which satisfy the criteria discussed in § 2 *d* originate almost exclusively from only four civilizations: ancient Babylon; ancient and medieval China; ancient and medieval Europe; and the medieval Arab world. For example, no early eclipse records appear to have been preserved from Egypt, India or Central America. The material from the four major sources will be discussed in turn, with examples. Full details, including analysis of individual observations will be published in Stephenson (1995). The numbers of observations from each source are listed in table 1.

(i) *Babylon*

At some time before 700 BC, Babylonian astronomers began to systematically predict and observe eclipses of both the Sun and Moon and this practice continued until perhaps as late as the 1st century AD. The city of Babylon ceased to be inhabited around AD 100 (Oates 1986, p. 143). However, no observational text more recent than about 50 BC has ever been found (Sachs & Schaumberger 1955). The latest datable texts from Babylon are almanacs, one of which contains predictions for AD 75 (Sachs 1975).

Ptolemy lists a few Babylonian timings of lunar eclipses in his *Almagest* (books IV–VI). However, these may well have been preselected by Ptolemy from a larger body of data to use as numerical examples (Toomer 1988). The times of observation are clearly not in their original form and Fotheringham (1915) was of the opinion that they had been 'very loosely reduced' by Hipparchus. Hence, we have not considered them further.

Fortunately, original Babylonian eclipse observations survive in large numbers. These are preserved in three types of text: astronomical diaries; so-called 'goal-year texts'; and texts devoted specifically to eclipses. Virtually all of the known records of this kind are now in the British Museum, having been recovered from the site of Babylon rather more than a century ago (Sachs 1974; Stephenson & Walker 1985). The various inscriptions are written on clay tablets using a cuneiform script, which, thanks to the painstaking work of scholars, is now well understood. Regrettably, most tablets are badly damaged and only about 10% of the original material is known to be extant.

Astronomical diaries provide the primary source of observations. These typically cover six or seven months and contain a day-to-day record of a wide variety of celestial events. The ultimate motive for observation was largely astrological.

Photographs, transliterations and translations of all of the datable diaries from 652 BC (the earliest known example) down to 165 BC have been published by Sachs & Hunger (1988, 1989). Publication of more recent diaries is currently in progress.

During the Seleucid period (late 4th century BC onwards), the Babylonian astronomers used the material in past diaries to compile both goal-year texts (designed to assist predictions for a selected future year) and texts devoted to specific phenomena (e.g., eclipses or planetary data). Because so many diaries are no longer extant, these later copies provide a valuable source of additional information.

Almost all of the observations preserved in the astronomical diaries and goal-year texts date from after 350 BC. Several compilations of eclipse records cover much the same period, though, fortunately, a number of eclipse texts containing records going back as far as about 700 BC are also extant. By chance, the surviving tablets of this kind are virtually all concerned with lunar eclipses; scarcely any texts specifically devoted to solar eclipse have come down to us, so that the available solar observations cover a much shorter time-span.

In an unpublished manuscript which he freely distributed, Huber (1973) provided transliterations and translations of numerous solar and lunar eclipse records which he extracted from the available diaries, goal-year texts and eclipse tables. Huber largely made use of the drawings of tablets published by Sachs & Schaumberger (1955). We have used this manuscript as our main source of Babylonian eclipse observations, supplementing this material from the works of Sachs & Hunger (1988, 1989) and also preprints of as yet unpublished transliterations and translations of diaries kindly supplied to us by Hunger.

The Babylonian calendar was luni-solar, the first month of each year beginning around the time of the spring equinox. Months began with the first visibility of the crescent Moon, while the day commenced at sunset. Extensive studies of the Babylonian calendar by Parker & Dubberstein (1956) have enabled dates between 626 BC and AD 75 to be converted accurately to the Julian equivalent. Babylonian dates are reported with consistently high precision, whether in original diaries or copied on secondary tablets.

Throughout the period covered by the texts, Babylonian astronomers systematically timed the interval between the onset of an eclipse and sunrise or sunset (whichever was nearer). In clear weather (such as prevails in this part of the world throughout the summer months), sunrise and sunset would be accurately defined moments, especially since the plain in which the site of Babylon lies is remarkably level. However, if cloud intervened when the Sun was near the horizon, uncertainties might be appreciable. The astronomers also measured the durations of individual eclipse phases and estimated the maximum degree of obscuration of the appropriate luminary, as well as noting if it rose or set eclipsed.

Probably some kind of clepsydra (water clock) was used for timing the various intervals. The standard units of time adopted were the *us* and *beru*, the former being equal to $\frac{1}{360}$ of the day and night and, thus, 4 min. Since this unit was the interval required for the celestial sphere to turn through 1° (Stephenson & Fatoohi 1994*b*), it is customary to translate *us* directly as degrees. The *beru* was equal to 30 *us*, or two hours. Eclipse magnitudes were estimated in *si* or fingers, 12 of which were equal to the solar or lunar diameter. This practice spread to the Greeks and thence to the medieval Arabs.

An illustrative example of Babylonian observations is provided by the record of the partial lunar eclipse of 11 April 80 BC. This is recorded on a tablet specifically devoted to the event and may be translated as follows (omitting superfluous details):

> 'Year 168 (Arsacid), that is year 232 (Seleucid), Arsaces, king of kings, which is in the time of king Orodes (I), month I, night of the 13th . . . 5° before μ Her culminated, lunar eclipse, beginning on the south-east side. In 20° of night it made 6 fingers. 7° of night duration of maximal phase, until it began to become bright. In 13° from south-east to north-west, 4 fingers lacking to brightness, it set [. . .] (Began) at 40° before sunrise' (BM 33562A, Obv. and Rev.; trans. Huber, pp. 75–76).

This account contains many of the characteristic features of a typical Babylonian eclipse record. It also gives the time of onset relative to the meridian transit of a selected culminating star. After about 250 BC, transits of about 30 stars known as *ziqpu* or culminating stars were frequently used for timing eclipses in addition to the more traditional sunrise or sunset methods (Schaumberger 1952). It is arguable that the identities of a few of the fainter culminating stars are doubtful, although most appear to be well established.

In the above example, it will be seen that the lunar eclipse began 40° before sunrise and the Moon was estimated to be half covered 20° later. No noticeable change in the degree of obscuration was observed for a further 7°, after which the phase began to decline. By the time the Moon set it was only one-third covered. Calculating the local time of sunrise at Babylon and allowing for the geographic longitude and the equation of time, the UT of sunrise corresponds to 2.67 h. Hence, the UT of first contact may be deduced as exactly 0.00 h. The result for first contact is closely supported by the measurement relative to the culmination of μ Her, which implies a UT of 0.05 h. Taking maximal phase as the middle of the period when no change in the degree of obscuration of the Moon was noticed, leads to a UT of 1.57 h for mid-eclipse, thus supplying a further independent measurement.

Two accounts of the total solar eclipse of 15 April 136 BC are preserved on separate Babylonian tablets. A goal-year text (BM 34034) asserts that the eclipse was total, while an astronomical diary (BM 45745) provides additional descriptive details.

> 1. '194 Seleucid, month XII/2, day 29, solar eclipse. When it began on the south-west side, in 18° of day in the morning it became entirely total (*til-ma til-ti gar-an*). (It began) at 24° after sunrise'. (BM 34034, Rev. 24–28; trans. Hunger, preprint).
>
> 2. '[. . .], day 29. At 24° after sunrise, solar eclipse; when it began on the south-west side [. . .] Venus, Mercury and the 'normal stars' were visible; Jupiter and Mars, which were in their period of disappearance, became visible in its eclipse [. . .]. It threw off (the shadow) from south-west to north-east. (Time interval of) 35° for onset, maximal phase and clearing. In its eclipse, the north wind which [. . .] was [set towards the west (?) blew . . .]'. (BM 45745, Rev. 13′–15′; trans. Hunger, preprint).

In the first example, month XII/2 denotes the intercalary 12th month. The date corresponds exactly to 15 April in 136 BC. Only the day of the month is preserved in the diary, but since numerous lunar and planetary observations are reported on the same tablet, the true date can be readily established by astronomical computation. Hence, it is quite certain that both texts refer to the same event, which is the only extant record of a total solar eclipse in Babylonian history.

Some of the time intervals measured relative to sunrise or sunset are extremely short. Thus, the solar eclipse of BC 321 was reported as beginning only 3° before sunset, while the lunar eclipse of 240 BC was said to begin only 3° before sunrise. In these and similar cases, clock drift – possibly rather serious for a primitive clepsydra when operating over several hours – would be minimal. In our analysis (see § 5 *b*) we have assigned double weight to all observations for which the time interval before or after sunrise or sunset was less than 25°. For timings expressed relative to the meridian transits of *ziqpu* stars (all short), we have assigned unit weight because of difficulties of identification in certain cases.

Of the 119 Babylonian contact timings for both lunar and solar eclipses which we have gathered together, more than 60 are of first contact while only about 20 are of last contact. The remainder relate to mid-eclipse and totality. Eight relate to estimates of the degree of obscuration at moonrise or moonset.

(ii) *China*

Although Chinese records of solar eclipses commence around 700 BC, there was little interest in reporting lunar eclipses until about AD 400, possibly because they were regarded as less serious omens than their solar counterparts. However, from these respective dates, the sequence of recorded events continues almost uninterrupted down to recent times. Most of these accounts are very brief, giving no more than the date of occurrence, and are thus of negligible value for the present purpose. However, a small percentage of the observations of both solar and lunar eclipses contain important details.

The principal sources of eclipse observations in Chinese history are the official dynastic histories. These have been reprinted many times and, except for the last (Qing) dynasty, all of the original reports have long since perished. A typical official history was often compiled fairly soon after the fall of the appropriate dynasty. In most of these works, eclipse observations are mainly to be found in two sections: a special treatise devoted to astronomy (including astrology), and the imperial annals. Additionally, the calendar treatises of a few histories also pay special attention to timed eclipses, while there are occasional qualitative accounts in the biographies of certain important personages. Eclipse observations cited in both the astronomical and calendrical treatises are probably nearly all derived from the records of the court astronomers, who maintained a regular watch for celestial phenomena of all kinds at the imperial observatory. Observations reported in the imperial annals and biographies are of more uncertain origin.

We have mainly confined our attention to eclipses cited in the treatises of the official dynastic histories. Some additional material has been extracted from chronicles and historical compendia such as the great *Wenxian Tungkao* (comprehensive history of civilization) compiled by Ma Duanlin around AD 1300. Recently, Beijing Observatory (1988) assembled an extensive list of celestial observations of all kinds (including eclipses) preserved in Chinese history. This work has proved to be a valuable secondary source.

All texts are written in classical Chinese. Following the unification of the Chinese empire in the 3rd century BC, the various scripts used in different parts of China were standardized. Since the 1st century AD there has been minimal change in the script of classical texts.

In expressing dates, years were numbered from the start of each reign period. Like the Babylonian calendar, the Chinese calendar was luni-solar. The first month of each year began roughly midway between the winter solstice and the spring equinox. From very ancient times, the Chinese also adopted a continuous 60-day cycle, independent of any astronomical parameter. This practice considerably facilitates date computation. Tables produced by various specialists (e.g., Hsueh & Ou-yang 1956) enable all dates from the beginning of the Han Dynasty (202 BC) to be accurately converted to the Julian calendar. Using these tables we have devised a computer program to facilitate date conversion. Like their Babylonian equivalents, Chinese dates are reported with consistently high accuracy.

Most ancient and medieval Chinese observations of solar eclipses merely record the occurrence of such an event on a specified day without any descriptive details; times or magnitudes are relatively rare. Occasionally, it is asserted that an eclipse was total but, especially in the more ancient works, further description is usually absent. Such is the case for the very early observations of 709, 601 and 549 BC recorded in the ancient chronicle known as the *Chunqiu* ('Spring and Autumn annals').

From about 200 BC onwards, occasional eclipses are also described as 'almost complete', but again most descriptions tend to be laconic. Both types of record possibly represent no more than rough summaries of the original observations. The astrological significance of an eclipse can be judged from the following quotation: 'Whenever an eclipse covers a small portion of the Sun the calamity it brings will be relatively small, but when it covers a large portion of the Sun the consequences will be much more serious' (from the astronomical treatise of the *Jin-shu*, a 7th century AD compilation translated by Ho Peng Yoke, 1966, p. 159). Hence, from time to time there may have been a tendency to exaggerate the magnitudes of large partial eclipses. As a result, we have selected from among the untimed observations only those reports which provide additional details: for instance, a total eclipse in which the appearance of stars is described, or a partial eclipse in which the unobscured portion of the Sun was said to be 'like a hook' (a common phrase). These would appear to be more careful summaries of the original accounts.

The following examples illustrate the quality of the available accounts of untimed solar eclipses.

1. AD 360 Aug 28 (capital Jiankang – modern Nanjing): 'Shengping reign period, 4th year, 8th month, day *xinchou* [38], the first day of the month. The Sun was eclipsed. It was not complete and like a hook' (*Songshu* (the official history of the Song Dynasty), ch. 34).

The above account is contained in the section devoted to eclipses observed in the previous Jin Dynasty. The *Jinshu* itself, the official history of the Jin Dynasty, notes that the eclipse was 'almost complete' without mentioning the resemblance to a hook. This eclipse was generally annular on the Earth's surface.

2. AD 454 Aug 10 (capital Jiankang): 'Xiaojian reign period, first year, 7th month, day *bingshen* [33], the first day of the month. The Sun was eclipsed; it was total; all the constellations (i.e. the lunar lodges) were brightly lit'. (*Songshu*, ch. 34).

In the above examples, each cyclical day number is given in square brackets after the name of the appropriate day.

Since accounts of eclipses cited in the treatises of the dynastic histories are probably taken from the records of the court astronomers, in most cases the place of observation was presumably the capital of the time. This would explain why most texts are silent regarding the place of observation; it would be regarded as understood. Occasionally we find records of the following form: ‘This eclipse was not seen at the capital but was reported from the provinces’. However, such instances – which probably reflect adverse weather at the capital – are normally rare, suggesting that there were few provincial observers.

From at least AD 400, it was the practice of Chinese astronomers to measure times of solar eclipses to the nearest *ke* (‘mark’), equal to $\frac{1}{100}$ of a day and night or 0.24 h. Previously, such times had usually been estimated to no better than the nearest double hour. At about the same date, lunar eclipses began to be timed to the nearest fifth of a *geng* (‘night watch’), but after about AD 1000, *ke* were preferred for all measurements. Although the units termed *ke* were of fixed length, the night watches varied with the seasons. Conventionally, the interval between dusk (2.5 marks or 0.6 h) after sunset and dawn (2.5 marks before sunrise) was divided into five equal watches, each of mean length rather more than two hours.

The standard timing device was a clepsydra, which was adjusted for seasonal variations in the lengths of the units when necessary (Needham *et al.* 1989). Such careful measurements were probably made almost exclusively at the capital, since few accurate instruments would be available in the provinces. Most recorded timings which are still preserved are restricted to two discrete periods: from AD 400–600 and again from AD 1000–1300. Presumably, many measurements in the intervening centuries have gone missing.

In all, we have assembled some 80 Chinese measurements of the contact times for both solar and lunar eclipses. These are roughly equally divided between first contact, mid-eclipse and last contact. We have rejected a few timings of solar eclipses since there are problems of interpretation.

Examples of Chinese solar and lunar eclipse timings are as follows. The first account notes the times for the various contacts, but the assertion that the eclipse was total at the capital of the time is probably of more importance.

3. AD 761 Aug 5 (capital Chang’an – modern Xi’an): ‘Shangyuan reign period, 2nd year, 7th month, day *guiwei* [20], the first day of the month. The Sun was eclipsed; the large stars were all seen. The Astronomer Royal, Chu Dan, reported (to the Emperor): ‘On day *guiwei* the Sun diminished. The loss began at 6 marks in the second half of the hour of *chen*. At 1 mark in the second half of the hour of *si* it was total. At 1 mark in the first half of the hour of *wu* it was restored to fullness. (The Sun) was 4° in (the lunar lodge) Zhang’ ’ (*Jiutangshu*, ch. 36).

The above measurements correspond to local times at Chang’an of approximately 9.56, 10.36 and 11.36 h, respectively, and, hence, UTs of 2.36, 3.16 and 4.16 h.

4. AD 434 Sep 5 (capital Jiankang): ‘Yuan-jia reign period, 11th year, 7th month, 16th day, full Moon. The Moon was eclipsed. . . . The Moon began to be eclipsed at the second call of the fourth watch, in the initial half of the hour of *chou*. The eclipse was total at the fourth call’ (*Song-shu*, ch. 12).

The times of the end of totality and of the restoration of brightness are not preserved. Calculations of the local time of sunset and the duration of night enable the measurements for the recorded phases of the eclipse to be reduced to local times of approximately 1.61 and 2.42 h, respectively, and thus UTs of 17.71 and 18.52 h (both on the previous calendar date, i.e. September 4).

Occasional texts state that the Sun or Moon rose or set while eclipsed and there are a very few estimates of the degree of obscuration of the luminary when on the horizon. Chinese astronomers usually expressed eclipse magnitudes in fifteenths of the solar or lunar diameter. Such records are illustrated by the following example.

> AD 513 Jun 25: 'Yanchang reign period, 2nd year, 4th month, day *jihai* [36]. The Sun was in *Ji* (lunar lodge) and the Moon in *Wei* (lunar lodge). (The Moon) rose from beneath the Earth with 3/15 remaining (covered). It gradually became full (i.e. fully illuminated)' (*Weishu* (the official history of the Northern Wei Dynasty), ch. 105).

(iii) *Europe*

Numerous untimed solar and lunar eclipses are recorded in the ancient Greek and Latin classics, but in virtually every case either the date, place of observation or eclipse magnitude (or a combination of these factors) is uncertain. Because of these deficiencies, scarcely any of the data selected by Fotheringham (1920*b*) in his much-quoted paper prove to be viable. Although Fotheringham obtained seemingly accurate results for the 'accelerations of the Sun and Moon', these were essentially determined from only three observations, none of which fulfil all of the criteria in § 2 *d*.

Ptolemy lists a small number of ancient Greek timings of lunar eclipses in his *Almagest* (books IV and VI). These were analysed by Fotheringham (1920*a*). Both date and place of observation are well established,

One of the most viable records of a solar eclipse from ancent Europe dates from 394 BC. This generally annular eclipse was recorded as partial by the contemporary Greek historian Xenophon:

> 'Next day he (Agesilaus) crossed the mountains of Achaea Phthiotis and for the future continued his march through friendly territory until he reached the confines of Boeotia. Here at the entrance of that territory, the Sun seemed to appear in a crescent shape' (Xenophon: *Hellenica*, IV, 3, 10; trans. Dakyns (1892), pp. 54).

Xenophon accompanied Agesilaus II of Sparta on his campaign and thus presumably witnessed the eclipse personally. From the careful description of the route followed, it is possible to deduce that the party was within about 10 km of Chaeroneia when the eclipse occurred. The year (the archonship of Diophantos) corresponds to 395/4 BC and the eclipse of 14 August 394 BC was the largest in the Eastern Mediterranean for several years. This is one fo the few ancient European solar observations that we have used.

Fortunately, medieval European solar reports are usually very reliable. From about AD 800 to 1500, chroniclers in towns and monasteries frequently noted the most striking celestial phenomena such as eclipses, comets, meteor showers and the *aurora borealis*. Dates in these works are usually accurate and since many chronicles were mainly concerned with local events, the place of observation can normally be taken as the town or monastery where the annalist lived. Descriptions of particularly large eclipses are often vivid and highly original; frequently it is reported that the chronicler himself witnessed the event. Accurate times are never given in annals, so the lunar eclipse observations are of no value for determining ΔT. However, many accounts of solar obscurations either carefully describe the complete disappearance of the Sun or affirm that a small part of the solar disc remained unobscured. More observations in these categories are preserved in medieval European annals than in any other early source.

A large number of medieval European chronicles have been published in their original language (which is usually Latin) by editors such as Muratori (1723–) and Pertz (1826–). About a century ago, Celoria (1877*a*, *b*) and Ginzel (1884*a*, *b*, 1918) made extensive searches of the published chronicles for accounts of solar eclipses. They were able to uncover numerous records which they quoted in their original languages. Newton (1972*b*), who gave valuable historical notes, provided translations of many of these records, though several of his quotations are incomplete. Wherever possible we have consulted the published chronicles and we have also made full use of the material compiled by Celoria and Ginzel.

Most medieval chroniclers used the Julian calendar, employing the terms calends, nones and ides. Seasonal hours (12 to the day and 12 to the night) were in common use, noon occurring at the 6th hour of the day. However, as noted above, times reported in chronicles were only crudely estimated.

Ambiguous terms such as *eclipsis universalis*, *eclipsis generalis* or *eclipsis in toto orbe* are frequently used in medieval annals. The most likely interpretation is that these were seen over a wide area rather than that the whole Sun was covered; such expressions are rarely accompanied by allusions to darkness and the other characteristic effects of totality. A report of the appearance of stars is itself not necessarily an indication of totality. For instance, stars were said to have been seen during the eclipses of AD 891, 990, 1147, 1191 and 1310. But all of these events were generally annular; in no case was more than 95% of the Sun obscured during the ring phase. Further, since none of these records allege annularity, only a very large partial eclipse may have been witnessed in each case. The occurrence of darkness is commonly noted in medieval texts. However, sensations of loss of daylight are highly subjective and we have restricted our attention to those accounts which either give a clear description of the complete disappearance of the Sun or assert that a portion of the solar disc remained unobscured even at greatest phase. It should be noted that unambiguous accounts of central annular eclipses are extremely rare.

The following examples indicate the quality of some of the better observations reported in medieval European chronicles.

> 1. AD 1147 Oct 26 (Brauweiler, Germany): ‘1147. On Sunday, the 7th day before the calends of November, a solar eclipse occurred at the 3rd hour and persisted until after the 6th. This eclipse stood fixed and motionless for a whole hour as noted on the clock (*horologium*).... During this hour a circle of diverse colours and spinning rapidly was said to be in the way’ (*Annales Brunwilarenses*; Pertz 1859, vol. 16, pp. 727).

Although the description is a little obscure, this is perhaps the most reliable allusion to the ring phase from medieval Europe.

> 2. AD 1178 Sep 13 (Vigeois, France): ‘In the year from the incarnation of

> our Lord 1178, on the 4th day of the week, the ides of September, the 28th day of the Moon, on a clear day, at about the 5th hour the Sun suffered an eclipse. Its disc began to be covered from the east (*sic*) until it became like a two- or three-day old Moon. The star Venus was seen to the north. After 6 the brightness returned from the east, in the order in which it was blackened, until the Sun was fully illuminated....' (*Ex Chronico Gaufredi Vosiensis*; Bouquet 1781).

Irradiation may well have considerably exaggerated the apparent width of the crescent.

> 3. AD 1241 Oct 6 (Stade, Germany): '1241.... There was an eclipse of the Sun on the octave of Michael, namely the day before the nones of October (i.e. October 6), on Sunday some time after midday. Stars appeared and the Sun was completely hidden from our sight. Yet the sky was so clear that no clouds appeared in the air'. (*Annales Stadenses*, Pertz (1859), vol. 16, pp. 368).

> 4. AD 1267 May 25 (Constantinople) 'At that time the Moon obscured the Sun when it was in the 4th part (degree) of Gemini, at the 3rd hour before midday on the 25th day of May in the year 6775 (Byzantine, i.e. AD 1267). It was a total eclipse of about 12 digits or points. Also, such darkness arose over the Earth at the time of mid-eclipse that many stars appeared' (*Nicephoras Gregoras Hist. Byzant.*, Lib. IV, cap. 8; Migne 1865).

The most important eclipse observations in the centuries immediately preceding the invention of the telescope are by the Jesuit astronomer Christopher Clavius. He observed a total eclipse in AD 1560 and one which was virtually complete in 1567 and although no times were measured, he provided detailed descriptions of them.

Clavius' description of the eclipse of AD 1567 April 9 (which immediately follows that of 1560 August 21) may be translated as follows.

> '... The other (eclipse) I saw in Rome in the year 1567 also about midday in which although the Moon was placed between my sight and the Sun it did not obscure the whole Sun as previously but (a thing which perhaps never before occurred at any other time) a certain narrow circle was left on the Sun, surrounding the whole of the Moon on all sides' (Clavius 1593).

This eclipse was annular-total, but in the vicinity of Rome several beads of sunlight would have remained visible even if the centres of the Sun and Moon were directly in line. The 'certain narrow circle' which Clavius described was presumably the inner corona.

(iv) *Arab dominions*

Medieval Arab records of eclipses are mainly to be found in two quite distinct sources: chronicles and astronomical treatises, the latter being termed *Zij*. Chronicles cover much the same period as the town and monastic annals of Europe (roughly from AD 800–1500). Further, the untimed and essentially qualitative descriptions of eclipses and other celestial phenomena contained in these works have much in common with those of European origin. Unfortunately, relatively few Arabic chronicles appear to have survived, so the number of extant reports of eclipses is correspondingly small. The few *Zij* which record eclipse observations cover only a relatively brief period – from around AD 800–1000. These compilations (notably the *Zij al-Kabir al-Hakimi* of ibn Yunus, dedicated to Caliph al-Hakim) contain many measurements of the times of both solar and lunar eclipses.

In both types of source, dates are normally expressed in terms of the Islamic lunar calendar. This assigns to every year 12 months, each of length 29 or 30 days. Hence, the Islamic year contains only 354 or 355 days, with the result that the beginning of the year continually retrogrades relative to the tropical year, making a full cycle of the seasons in about 33 years. Years on this scheme (designated AH) are numbered from the *Hijra*, the migration of Muhammad from Mecca to Medina in AD 622. Tables for conversion of Muslim dates to the Julian or Gregorian Calendar have been produced by Freeman-Grenville (1977).

The lunar eclipse observations reported in Arab chronicles are probably too crude to be of value for the present purpose. On the other hand, several total solar eclipses are reported in graphic detail. Among these may be cited the following entry in the chronicle of ibn al-Jawzi on a date corresponding to AD 1061 June 20.

> (453 AH). ‘On Wednesday, when two nights remained to the completion of Jumada al-Aula (the 5th month), two hours after daybreak, the Sun was eclipsed totally. There was darkness and the birds fell while flying. The astrologers claimed that one-sixth of the Sun should have remained (uneclipsed) but nothing of it did so. The Sun reappeared after four hours and a fraction. The eclipse was not in the whole of the Sun in places other than Baghdad and its provinces’ (trans. Said *et al.* 1989).

As well as giving the correct date, this account is quite definite with regard to the complete disappearance of the Sun and furthermore it clearly specifies the place of observation.

The work by al-Biruni known as *al-Qanun al-Mas’udi* (‘the Masudic Canon’) contains an interesting observation of an annular solar eclipse as seen from Nisapur (Neyshabur) in Iran. He relates that it was seen early in the morning of Tuesday 29 Ramadan, in 259 AH by Abu al-’Abbas al-Iranshahri. The date corresponds to AD 873 July 28.

> ‘He (al-Iranshahri) mentioned that the body of the Moon was in the middle of the body of the Sun so that the light from the remaining portion of the Sun surrounded it uneclipsed; and it is clear from this that the diameter of the Sun exceeds in view that of the Moon’ (trans. Goldstein 1979).

Evidently al-Biruni was unaware of any accounts of total eclipses. The above report is important for its assertion of a central annular eclipse. However, it only gives an approximate indication of the time of day.

Rather than use a clepsydra, medieval Muslim astronomers preferred to measure eclipse times indirectly by determining altitudes using a quadrant or astrolabe, afterwards reducing their results to local time. These instruments were probably much more accurate than the crude timing devices of the period. The following examples from the *Zij* of ibn Yunus indicate the care with which Muslim astronomers often measured the times of both solar and lunar eclipses:

1. AD 927 Sep 14 (Baghdad: lunar eclipse reported by ibn Amajur): (315

> AH, Friday – month and day of month not stated). 'This eclipse was observed by my son Abu al-Hasan. Altitude of the star al-Shi'ra al-Yamaniya (Sirius: α CMa) at the start, 31° in the east; revolution of the sphere between sunset and the start of the eclipse (presumably determined with the astrolabe), approximately 142°, 9 h 52 min equal hours, which is 10 unequal hours. Estimated magnitude of the eclipse, more than one quarter and less than a third, approximately $3\frac{1}{2}$ digits' (trans. Said & Stephenson 1991).
>
> 2. AD 977 Dec 13 (Cairo: solar eclipse observed by ibn Yunus): (367 AH, Rabi I, day 28). 'We assembled in order to observe this eclipse several scholars at Qarafa (a district of Cairo) in the mosque of al-Maghribi.... Everyone waited for the start of the eclipse; when it began to become visible the altitude of the Sun was greater than 15 and less than 16°. Those present agreed that the Sun was eclipsed by about 8 digits of diameter.... The Sun cleared completely when its altitude, as I estimated, was greater than 33° by about one-third of a degree, everyone being in agreement for the completion of clearance (i.e. the end)' (trans. Said, personal communication).

Ibn Amajur, as quoted by ibn Yunus, also provides evidence that medieval Arab astronomers viewed solar eclipses by reflection in water in order to reduce the glare of the Sun. It is not known which method was adopted by the Babylonians or Chinese.

Most of the 43 Arab timings are of first and last contact and these occur in roughly equal number for both solar and lunar eclipses. In all but nine, the original altitude measurement is still preserved. Under these circumstances, we have converted this measurement to the equivalent local time rather than rely on the observer's own reduction.

(*b*) *Lunar occultations AD 1620–1955.5*

Over 53 000 timings of lunar occultations of stars in the period AD 1620–1955 are catalogued by Morrison (1978) and Morrison *et al.* (1981). In the early part of this period, the individual observations are given to a precision of 1 s. From AD 1800 approximately, the precision is generally 0.1 s. These occultation observations enable variations in UT to be traced in detail on a timescale upward of a year. Details of the analyses of the observations from AD 1830–1955.5 can be found in Jordi *et al.* (1994).

4. Reduction to ΔT

Eight different categories of eclipse observation which are, in principle, of value in the determination of ΔT are listed in § 2 *e*. Historical examples are cited in § 3. In this section, the determination of ΔT from each type of observation is discussed.

(*a*) *Pre-telescopic solar observations*

(i) *Untimed total and annular solar eclipses*

Provided that a solar eclipse is known to have been total or annular on a definite date and at a specified place, measurement of the time of day when it occurred is unnecessary. The umbral shadow sweeps out a narrow zone across the Earth's surface up to about 400 km wide. The effect of increasing ΔT is to displace the calculated zone towards the east (relative to the Earth's surface), and *vice versa*. If the track of central eclipse is inclined at a fairly large angle to the equator in the vicinity of the place of observation, only a rather small range of ΔT – typically about 1000 s in width – will satisfy the record. Since the onset of totality or annularity is usually well defined, the two ΔT limits are very sharp. In particular, if the zone of totality or annularity is narrow (i.e. the apparent diameters of Sun and Moon are nearly equal), the observation will be especially valuable since the indicated range of ΔT will be extremely small. However, if the track of the umbra runs almost parallel to the equator, the required range of ΔT may be so wide as to render the observation redundant. The local geographical circumstances are thus of considerable importance.

We have used an iterative procedure to derive the range of ΔT satisfying each observation of a total or annular eclipse. This involves deducing, in turn, the values of ΔT which would cause each edge of the central zone to touch the place of observation. Since estimates of the very brief duration of totality or annularity are so rarely reported in ancient and medieval texts, it is seldom possible to derive narrower ΔT limits than those obtained in this way.

As an illustration, the total eclipse observed at Babylon in 136 BC may be cited (see § 3 *a* (i)). This is by far the most reliable ancient record of a total obscuration of the Sun, and in the neighbourhood of Babylon the track of totality was inclined to the equator at a large angle (some 60°). Computations which assume a fixed day length of 86 400 s SI make the track of totality pass far to the west of Babylon. In order for the southern edge of the umbral shadow to actually touch Babylon, a value of ΔT of 11 210 s (3.11 h) would be needed. For the northern edge to cross the site, the corresponding figure would be 12 140 s (3.37 h). Hence, to satisfy the observation, only values of ΔT between these two limits are acceptable. Similar remarks apply to other observations. As noted above, the value of some otherwise highly reliable records of totality are rejected for the present purpose since the track of the umbra ran almost parallel to the equator. Unfortunately, such was the case in Italy at the eclipse of AD 1239, for which unusually detailed observations of totality were reported at a variety of sites.

(ii) *Untimed large partial solar eclipses*

Although many observations of solar eclipses specifically state that they were not total, affirming instead that the Sun was reduced to a thin crescent, it is very rare for a record to describe the direction in which the cusps were pointing. Hence, in almost every case, all that can be assumed is that the path of totality passed to the north or south of the place of observation, whereas even a simple description could have eliminated one of the options. Reduction of an untimed record of a partial eclipse (in which totality or annularity is specifically denied) proceeds very much as for totality, but in this case a range of ΔT is excluded. If this zone of avoidance is very narrow (corresponding to an eclipse in which the apparent diameters of Sun and Moon were nearly equal), the record may tell us very little: in most cases, virtually any reasonable value of ΔT on either side of the excluded range is acceptable since estimates of the magnitude of a large partial eclipse – if recorded at all – are often very crude. On the other hand, if the projected zone of totality is wide, it is usually possible to discard the solution space for ΔT on one side of totality by reference to the mutual consistency of the solutions from other contemporaneous eclipses.

The very large partial eclipse of AD 1178, reported from Vigeois in France (see § 3 *a* (iii), example 2), provides a useful example. For any value of ΔT between 1140 and 2320 s, this eclipse would have been total at Vigeois. Hence, to satisfy the observation that the eclipse was not total, either $\Delta T < 1140$ s or > 2320 s. Values of ΔT far beyond these limits would still produce a large partial eclipse, so only the excluded range is significant.

(iii) *Solar eclipse magnitudes*

The computed magnitude of a solar eclipse is a weak function of ΔT. In order to assess the potential of this method, we have investigated the discrepancies between recorded estimates of *lunar* eclipse magnitudes and their computed equivalents (which are independent of ΔT). We conclude that mean errors, whether for Babylonian, Chinese or Arab observations, are close to 10% (see also Stephenson & Fatoohi 1994*a*). For a solar eclipse, a change in the value of ΔT by some 40 min would be needed to produce such an alteration in magnitude. Thus, such observations give poor resolution. Further, it is seldom stated whether the upper or lower portion of the Sun was covered, so that each estimate of magnitude leads to two discrete values for ΔT, typically of the order of 6 h apart. Since preserved estimates of solar eclipse magnitude are relatively few in number, we have not considered them further in this analysis.

(iv) *Observations that the Sun rose or set eclipsed*

Occasionally it is stated that the Sun rose or set while partly obscured. Such an observation is satisfied by a range of values of ΔT – rather similar to the case of a total solar eclipse, although the limits are much further apart and less well-defined. In certain cases it can only be established from the record that the Sun reached the horizon at some time between first and last contact. Thus the range of ΔT can span up to 2 h for an eclipse of large magnitude. Sometimes it is possible to decide from the text whether the observation was made while the phase was growing or diminishing, thus halving the derived range of ΔT.

(v) *Timed solar eclipse contacts*

Solar eclipse timings which are preserved in ancient and medieval history usually relate to both the beginning and end of the event (first and last contacts) and also maximum phase. The various measurements, whether made directly (usually with the aid of a water clock) or indirectly (expressed in terms of solar altitudes) are readily reducible from local apparent time to UT by adjusting for geographic longitude and the equation of time. Thus by computing the TT of the appropriate phase, a result for ΔT may be obtained by direct subtraction.

The calculation is not straightforward though, because the curvature of the eclipse shadow on the Earth's surface renders the calculation of TT nonlinear. In practice, it is calculated iteratively using successively improved values of ΔT.

Most early timings were very crude (typically to no better than the nearest 10 or 20 min). Hence, it makes little difference whether actual contacts or the usually less well-defined maximum phase are utilized. In principle, last contacts should normally be the most accurate since the observer merely needs to watch the indentation at the western limb of the Sun gradually disappear. However, among ancient and medieval astronomers it was common practice to roughly calculate in advance the time of onset for an eclipse so that observers – most of whom would be very experienced – would have some idea of when to watch for first contact. The fact that on many occasions the solar altitude at the start of an eclipse was measured to the nearest half degree suggests that a careful watch was usually maintained.

On rare occasions, Arab astronomers made allowance for the delay in detecting the start of an eclipse if they had failed to keep a systematic watch of the Sun, possibly, for example, on account of intermittent cloud. Thus, ibn Yunus stated that in AD 1004, the solar altitude at Cairo was 16.5° in the west when the eclipse was first noticed, but he estimated that the true beginning occurred some 10 min before when the altitude was 18.5°. Ibn Yunus was experienced in observing eclipses and his empirical correction should be fairly reliable.

(b) *Pre-telescopic lunar observations*

(i) *Estimates of the degree of obscuration of the Moon at moonrise or moonset*

There appear to be scarcely any corresponding observations for solar eclipses which can be regarded as viable so that we have not devoted a separate section to their analysis. However, several careful estimates of the fraction of the Moon which was enveloped in the Earth's shadow when it rose or set are preserved. These estimates are usually quoted to better than 10%. Since the phase of a central eclipse changes by such an amount in only about 6 or 7 min, it would appear that an observation of this type might compete fairly well with a carefully timed contact.

When the Moon is rising or setting, atmospheric refraction is maximal. Nevertheless, distortion of the lunar limb is not necessarily critical. Photographs which we have inspected showing the eclipsed Sun on the horizon (few seem available for the eclipsed Moon in a similar situation) lead us to infer that on most occasions a reasonable estimate of the proportion of the disc in shadow would be possible, whether for the Sun or the Moon. In any case, if the limb of the luminary were very distorted, an observer in the ancient or medieval world might well feel that such an exercise was pointless. Individual results for ΔT obtained from these observations prove to be remarkably self-consistent (see below).

When an estimate of the extent to which the Moon was covered at moonrise or moonset is recorded, it would seem best to assume that the whole of the disc was just visible. Allowing for horizontal refraction, this corresponds to a zenith distance for the Moon's centre of 90.3°. In deducing the local time of moonrise and moonset, it is necessary to make a correction for parallax.

(ii) *Observations that the Moon rose or set eclipsed to an unknown degree*

As well as partial eclipses of unspecified magnitude, in this category are have included instances where it is recorded that the Moon was said to be totally eclipsed when on the horizon. Once again, analysis follows much the same course as for a solar eclipse, a range of ΔT being indicated. If an eclipse was generally total, yet the text states that the Moon rose or set while partly covered, it is usually possible to decide from the record whether the Moon reached the horizon before or after the total phase. Otherwise, two separate solutions for ΔT are obtained separated by up to 1 h 40 min. When the totally eclipsed Moon is on the horizon, it is invisible to the unaided eye.

(iii) *Timed lunar-eclipse contacts*

More timings of lunar than solar eclipses have survived, mainly because of the large excess of Babylonian observations in this category. As well as first and last contacts, many measurements of the times of beginning and end of totality (second and third contact), and also of maximum phase for partial eclipses, are preserved. As noted above, times might be measured directly or indirectly. However, once these diverse measurements are reduced to UT, computation of the TT of the appropriate phase leads to a result for ΔT by direct subtraction. There is no need for an iterative solution (unlike the case of a solar eclipse), since at any given moment the local circumstances of a lunar eclipse are virtually identical over the entire hemisphere of the Earth's surface from which the Moon is visible. In analysing all observations of the umbral contacts for lunar eclipses, we have applied the standard increment of 2% to the radius of the Earth's shadow to allow approximately for the effect of the atmosphere.

5. Analysis

The values for ΔT, which we have deduced from the various ancient and medieval observations, are the raw observables of this analysis. The main objective is to fit a smooth curve through them and thereby measure the historical variations of the UT scale. The first derivative along this curve is a measure of the fluctuations in the l.o.d.

The ΔT values are analysed in the two broad groups: untimed and timed. These two groups contain completely independent datasets, and this fact is used to test the reliability of the curves which are fitted to them. We start by testing the simplest model for the departure of the Earth's rotation from uniformity, i.e. a constant deceleration under the influence of tidal friction. This will produce a simple parabolic divergence of UT from TT.

Before we begin the mathematical analysis of the ΔT values, however, we need to accommodate the discontinuity of the BC/AD dating system. In the earlier part of this paper we expressed all eclipse dates in terms of BC or AD. This seemed most reasonable to us since the records are derived from historical sources. However, for astronomical purposes it is customary to replace BC and AD by $-$ or $+$ signs, partly for ease of numeration, but especially because the BC/AD system is discontinuous; there is no year zero, so that 1 BC is immediately followed by AD 1. Conventionally, AD and $+$ dates are identical but BC and $-$ dates differ by one. Thus BC 311 $= -310$ and so on. In the remainder of this paper, all dates are expressed using $-$ or $+$ signs.

(*a*) *Constant decelerative component of Earth's spin*

The results for ΔT obtained from the untimed data are plotted in figure 2. The data for the period $+1500$ to $+2000$ are shown inset on a greater scale. The untimed data produce ranges (solution space) of ΔT, anywhere in which the actual value is equally likely to lie.

A total eclipse usually defines a relatively narrow solution space for ΔT, with sharp upper and lower boundaries. Whereas, a partial eclipse defines the inverse of this, with a solution space filling the whole range of ΔT, except for a narrow gap in which the solution cannot lie because there the eclipse would be total. In all but one ($+1004$) of the ten partial solar eclipses, the solution space on one side of the belt of totality is redundant. These redundant solutions are not shown in figure 2.

Untimed observations which indicate that the Moon (or rarely the Sun) rose or set while eclipsed produce similar limits to those for total or annular solar eclipses, but often with much wider limits. Of the 34 observations of this type, which are collectively labelled lunar umbral eclipses, only 14 contribute useful constraints on the solution space for ΔT, and only these are plotted in figure 2.

The modern continuous curve derived from occultations is shown inset on a greater scale. This curve is taken from our earlier analysis (Stephenson & Morrison 1984).

For the sake of clarity and for discussion later, we reproduce the data plotted in figure 2 on increased scales for the periods −700 to +500 (figure 3) and +400 to +1600 (figure 4). Figure 4 also shows points resulting from timed data which are required for the discussion later.

In conformity with the model of constant deceleration, we have fitted a parabola to the data in figure 2 which satisfies the greatest number of boundary conditions set by the eclipses, and which has its cusp somewhere near the beginning of the 19th century AD, as explained in § 2 *a*. This parabola, which is drawn as a dashed curve in figures 2–5, has the equation

$$\Delta\mathrm{T} = (-20) + 31t^2\ \mathrm{s},$$

where t is measured in centuries from the cusp of the parabola which lies at the epoch +1820. The exact value of the constant (−20) is uncertain, since a value in the range −30 to −10 will bring the parabola into tolerable agreement with ΔT after +1600. The data before +1600 cannot provide resolution as good as this.

While, in general, this parabola is a remarkably good fit to most of the data, including the modern data from +1600 to +1990, as is evident from figures 3 and 4, it does not satisfy the very reliable boundary conditions of several observations. These relate to the total or annular eclipses of +454 (see § 3 *a* (ii), example 2), +761 (see § 3 *a* (ii), example 3), +1147 (see § 3 *a* (iii), example 1) +1221 (see Stephenson & Yau 1992) and +1267 (see § 3 *a* (iii), example 4) and the partial eclipse of +1178 (see § 3 *a* (ii), example 2). For this reason, we eventually reject this solution. Nevertheless, it serves to show that the parabola

$$\Delta\mathrm{T}_{\mathrm{tidal}} = (-20) + (42 \pm 2)t^2\ \mathrm{s},$$

expected on the basis of tidal friction alone (see § 2 *c*), is certainly not a viable solution.

Figure 2. Plot of the results for ΔT −700 to +1990 derived from untimed data. The inset (on a scale 25 times greater) includes the continuous curve derived from lunar occultations in the period AD 1620–1955.5, and the difference TAI − UT1 + 32.184 s from 1955.5–1990. The dotted curves are the 1σ limits of the parabolic divergence of UT from TT expected on the basis of tidal friction. The dashed curve is the best-fitting parabola $31t^2$ s (see § 5 *a*). The solid curve is fitted using cubic splines (see § 5 *b*). This curve, the best-fitting parabola, and the parabola expected on the basis of tidal friction are continued in the inset. Note that the vertical lines are not error bars, but solution space, i.e. anywhere in which the actual value of ΔT is equally likely to lie. Arrowheads denote that the solution space extends several thousand seconds (see text for details).

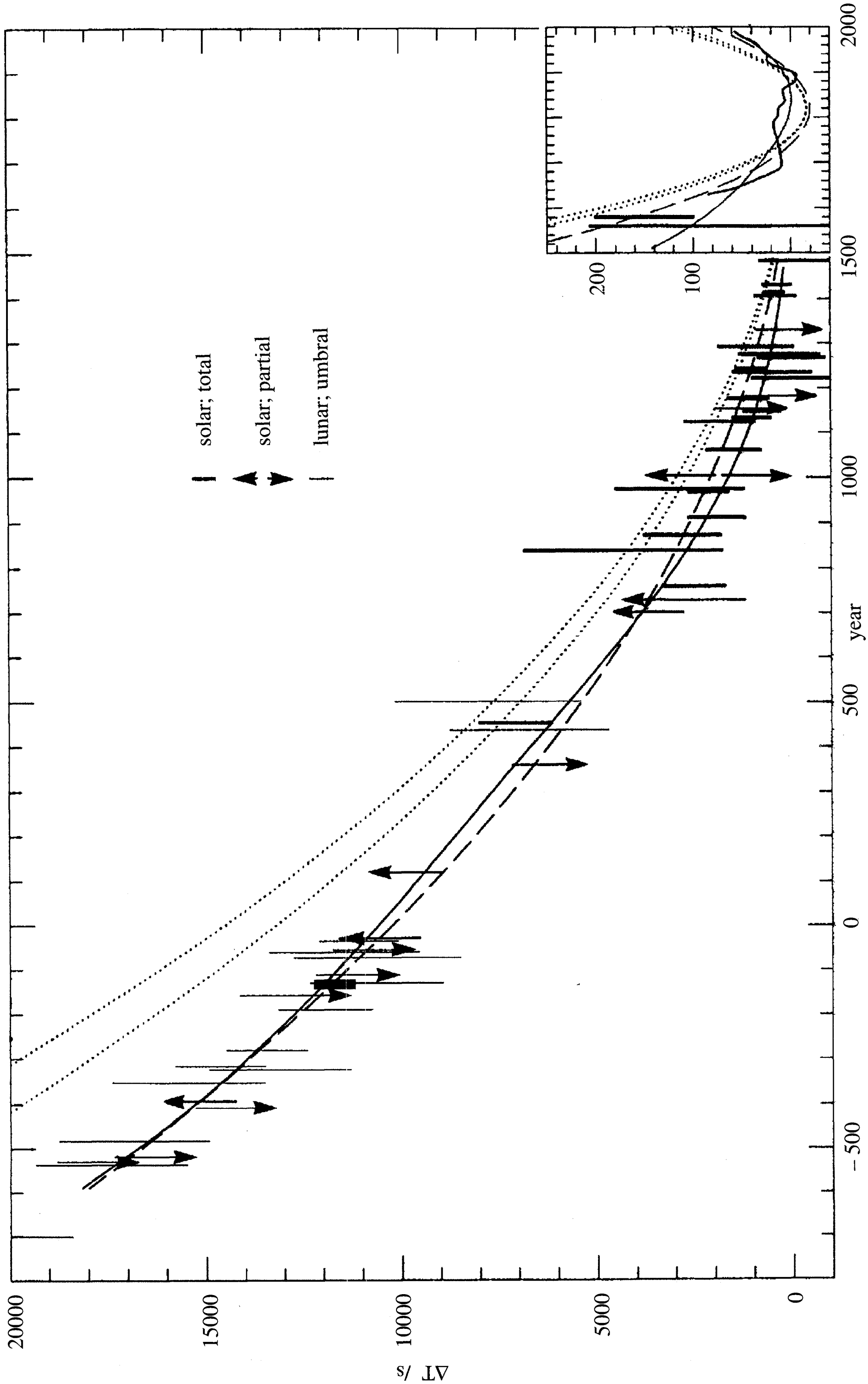

Figure 2. For description see opposite.

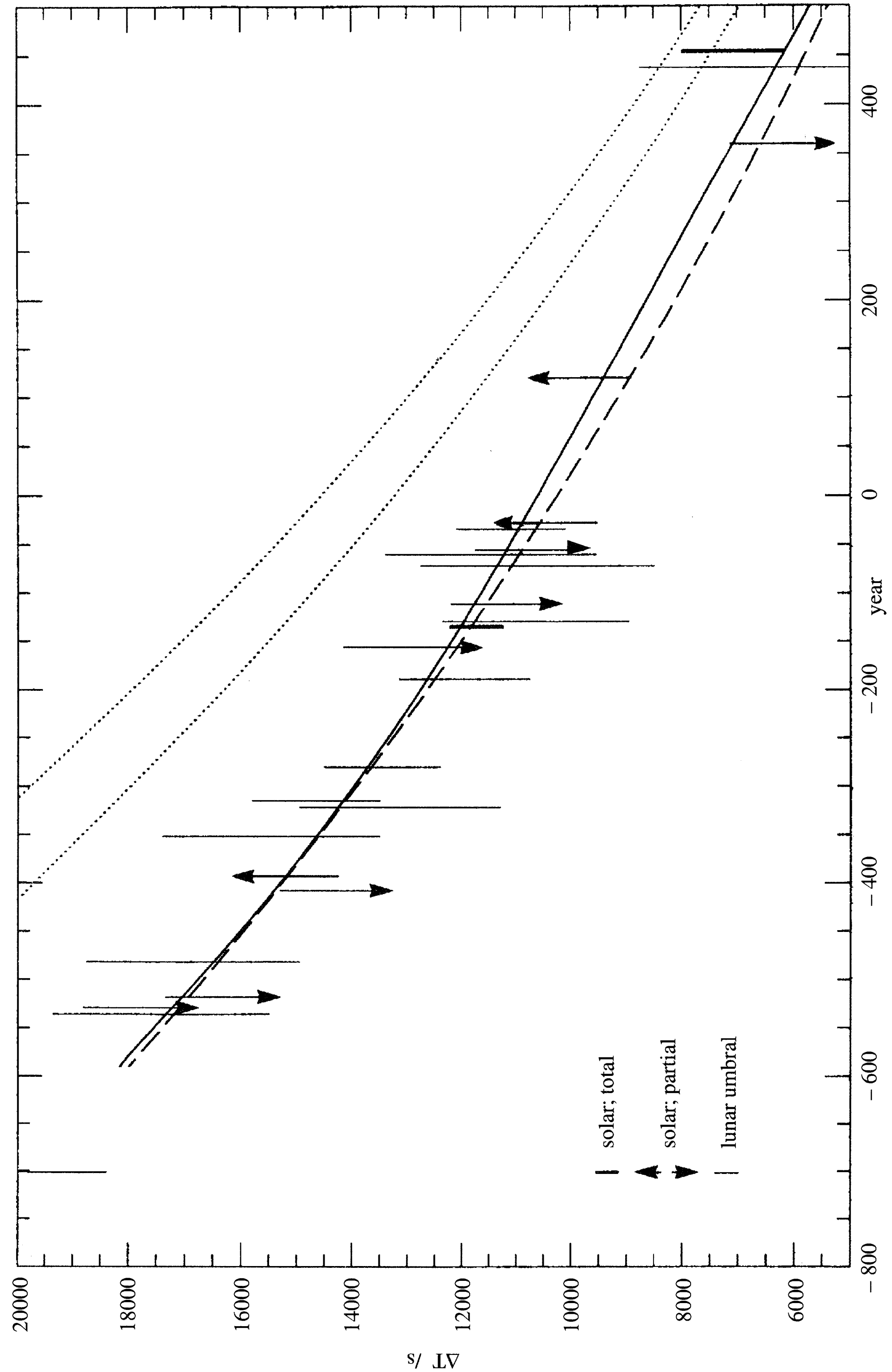

Figure 3. The data shown in figure 2 for the period −700 to +500 plotted on a larger scale.

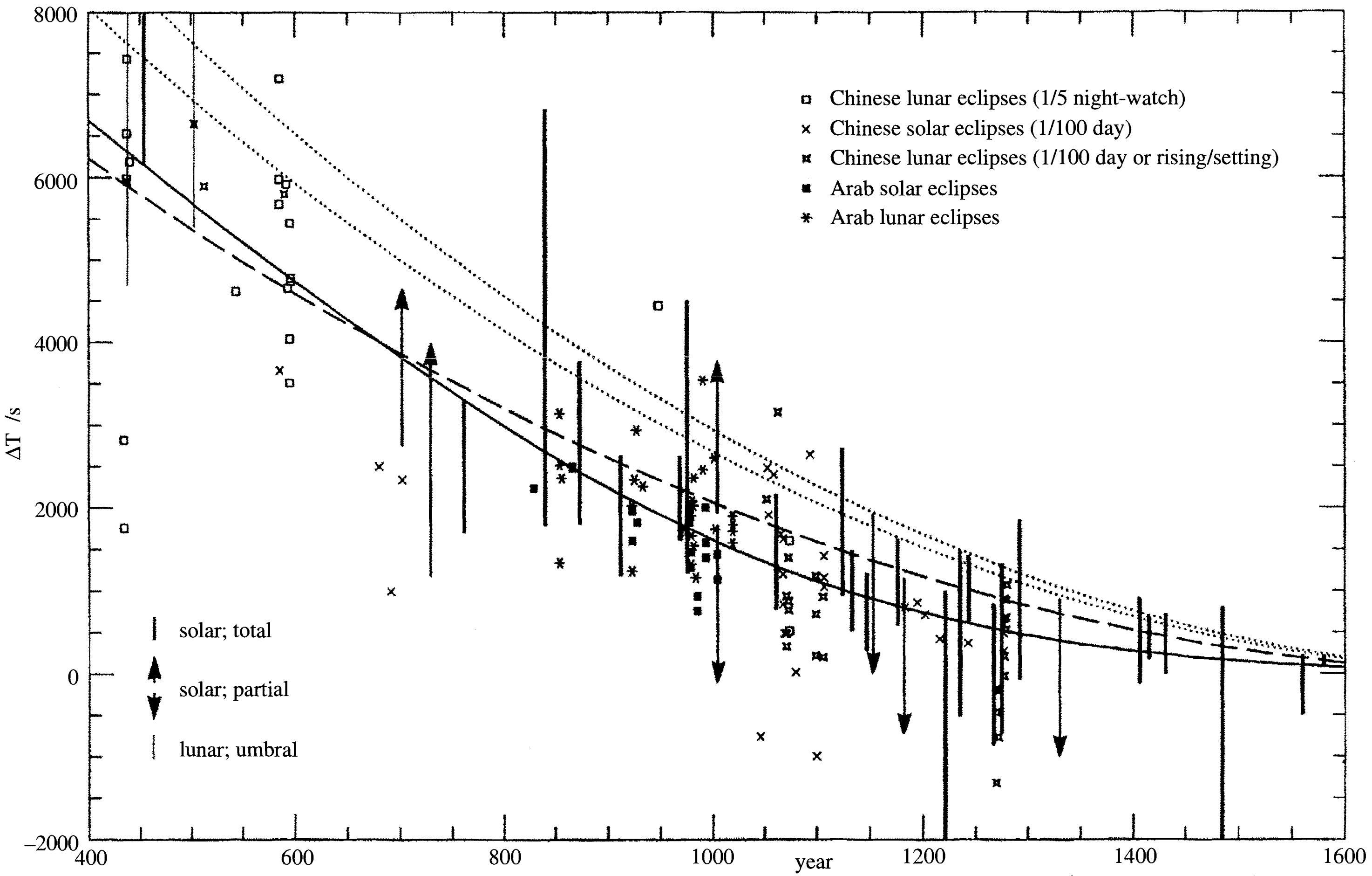

Figure 4. The data shown in figure 2 for the period +400 to +1600 plotted on a larger scale. The figure also includes the results for timed data for the same period.

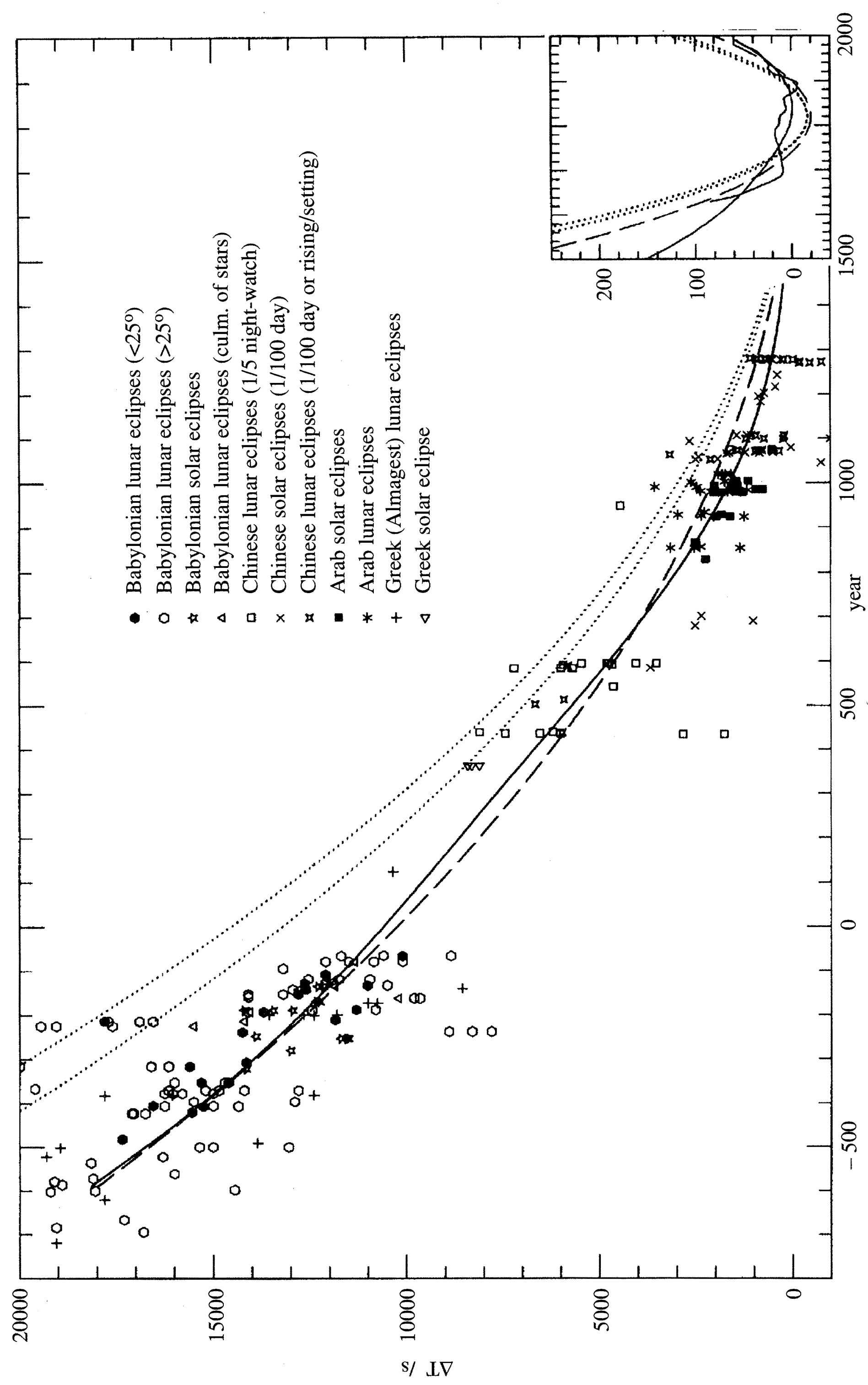

Figure 5. Plot of the results for ΔT −700 to +1990 derived from timed data. The same curves are shown here as in figures 2–4.

We now consider the timed data before +1600 which, in contrast to the untimed data, provide discrete values of $\Delta\mathrm{T}$. These discrete values, together with the same parabolas shown in figure 2, are plotted in figure 5 (and figure 4). Again, it is obvious that the tidal parabola alone is not a viable solution. The parabola fitted to the untimed data (shown dashed), while being a reasonably good fit overall, is not a good fit to the points around +1000 (see figure 4). Most of the points lie below the parabola. Steeper parabolas can be found which will fit the timed data better around +1000; but they will not fit the modern occultation curve between +1600 and the present. For example, the parabola (not shown in the figures to avoid confusion)

$$\Delta\mathrm{T} = (-20) + 35\tau^2 \text{ s},$$

with τ measured from the cusp at +1735 in centuries, is a good fit to the data before the year 0, and is a better fit than $+31t^2$ to the timed data circa +1000, though most of the points still lie below this steeper parabola. However, it does not remotely resemble the $\Delta\mathrm{T}$ curve between +1600 and the present, and it does not satisfy the requirement that the cusp should lie near the beginning of the 19th century because of the considerations discussed in § 2 *a*. If the parabola is forced to satisfy this requirement by translating it sideways, such that the cusp falls at +1820, the value of the parabola increases by ~ 1100 s at −135 and ~ 1350 s at −500, bringing it well above the upper limit of the very dependable eclipse of −135 (see § 3 *a* (i), example 1) and above most of the timed Babylonian data.

One parabola will not adequately represent all the data.

It is clear from the two independent datasets that the deceleration in the Earth's spin from tidal friction alone is too great, and, while the tidal retardation should be constant over the past 2700 years, the data indicates some departure from constancy. The inescapable conclusion is that as well as a constant tidal deceleration, there is a non-tidal accelerative component in the Earth's spin and this is variable with time. We now investigate this variable component by fitting a more flexible curve than a parabola to the data.

Before doing this we set bounds on our result of $+31 \text{ s cy}^{-2}$ for the coefficient of t^2 using the lower and upper limits of the Babylonian total solar eclipse of −135. This puts bounds of $+29.3 \text{ s cy}^{-2}$ and $+31.8 \text{ s cy}^{-2}$ on the coefficient, although, as we shall demonstrate in the next section, the Babylonian timed data indicate a solution closer to the upper bound. Hence, our preferred solution is $+31.0t^2$ to which we assign the range $\pm 0.9t^2$. This is equivalent to a change in the l.o.d. of $(+1.70 \pm 0.05) \text{ ms cy}^{-1}$.

(*b*) *Variable accelerative component of Earth's spin*

Given the distribution of the pre-telescopic data, and the closeness of fit of the parabola $31t^2$, it is only feasible to look for small departures from a constant acceleration on a timescale of hundreds of years. So, in fitting a more flexible curve to the data, it should be tightly constrained, otherwise spurious short-term fluctuations will result.

The method of curve-fitting that we chose was that of cubic splines. Cubic splines consist of a number of cubic polynomial segments (fitted by least-squares) joined end to end with continuity in first and second derivatives at the joins (knots). Continuity in derivatives provides a realistic model for changes in the l.o.d., and by restricting the number of polynomial segments (knots), we can readily constrain the degrees of freedom.

Fitting cubic splines with knots at the epochs −500, +300, +1000, +1700 and +1990 was found to be the best approach, conversant with the principle of economy of degrees of freedom. The splines were fitted to the data plotted in figures 2 and 5 in the following way: the points for the Babylonian lunar eclipses ($< 25°$) (see § 3 *a* (i)), and the medieval Arab solar and lunar eclipses – which were observed with exceptional care – (see § 3 *a* (iv)) were given twice the weight of the other classes of timed observations, apart from the modern occultation data which were represented by normal points of high weight at 50-year intervals between +1650 to +1900, and 25-year intervals thereafter. To this set of weighted points, critical limits imposed by the boundaries of the following total solar eclipses were added with high weight: −393, −135, +454, +761, +1241 and +1567. The resulting cubic-spline curve is plotted in figures 2–5 as a continuous curve.

The spline curve satisfies all the constraints in figure 2 imposed by the limits of the untimed solar eclipses, and all those of the lunar eclipses bar one – that of −408, which it narrowly misses (see figure 3). This exception is a Babylonian observation of a lunar eclipse which occurred near moonrise when at least part of the eclipse was still visible (§ 4 *b* (ii)). The text implies that the eclipse was visible for at least 8°, which leads to a value for $\Delta\mathrm{T}$ of less than 15 400 s. However, a faulty measurement or a scribal error could be responsible for the discrepancy, and we do not regard this single instance as serious grounds for altering the spline curve.

There are two periods where the spline departs significantly from the dashed parabola: around +400 and +1100. In the first of these (see figure 2), the main constraint on the spline is the lower boundary of the total solar eclipse of +454. The solar eclipse of +454 was recorded as total in the treatise of the *Songshu*, where it is also related that during the darkness of totality ‘all the constellations were brightly lit’ (see § 3 *a* (ii), example 2). There can thus be little doubt that the Sun was seen to be totally obscured. Since there is nothing to suggest that the observation was reported from outside the capital of Jiankang (e.g., on account of unfavourable weather there), it is reasonable to suppose that this was indeed the place of observation. As noted above, such treatises were compiled from the records of the court astronomers. We, therefore, regard the lower boundary of +454 as firm.

Note that the spline cannot pass much above the lower boundary of +454 without infringing the partial eclipse of +360. The account of this latter event in the *Songshu* treatise notes that, at maximum, the Sun had the appearance of a hook (see § 3 *a* (ii), example 1). This clearly indicates that the observers did not witness the annular phase, but only saw a large partial eclipse. We conclude that the limits for +360 are reliable.

The timed data around +400 do not themselves provide a strong constraint on the spline curve (see figure 5). However, such as they are, they favour a solution above the dashed parabola, especially when one considers that the two low points are obtained from two phases of the same Chinese lunar eclipse – that of +434 – and thus are not independent. Possibly, on this occasion, the clepsydra was malfunctioning or careless readings were taken.

In the second period around +1100, where the spline differs from the parabola, as well as the evidence of four infringements (+1147, +1178, +1221, +1267 in figure 4) of the untimed data by the dashed parabola, there is also strong evidence from the two independent sets of timed data from the Arab and Chinese

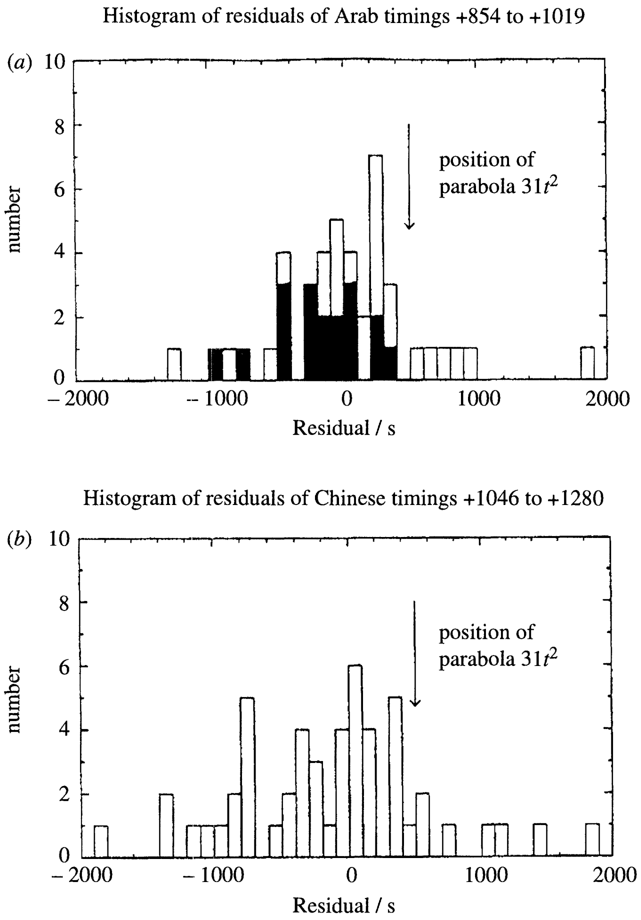

Figure 6. Histogram of the residuals of the timed data from the spline curve for (*a*) the Arab data +854 to +1019, and (*b*) the Chinese data +1046 to +1280. The arrow indicates the mean position of the best-fitting parabola $31t^2$ (see figure 4). The shaded part in (*a*) is the histogram of the residuals for the Arab solar eclipse timings.

civilizations. Histograms are plotted in figure 6 of the residuals from the spline curve for the Arab timings (+854 to +1019) and the Chinese timings (+1046 to +1280). The arrow indicates the position of the dashed parabola $31t^2$. The parabola is clearly at variance with both datasets and shows the same bias of about 500 s (~ 8 min) in each case. The value of ΔT derived from the spline curve is less than that from the parabola, which means that the UT is greater for the spline curve. So, if we assume for the sake of argument that the parabola is correct, could there have been a systematic delay of about 8 min in the Arab and Chinese timings of solar and lunar eclipses?

This seems extremely unlikely to us on three counts.

1. The Arabs and Chinese astronomers used entirely different methods of timing.

2. The progress of the indentation on the lunar or solar limb is probably discernible to the unaided eye after considerably less than 8 min (for example, in a central eclipse, the obscured portion of the diameter subtends 1 arcmin after no

more than about 2 min). The great medieval astronomer al-Biruni, who died in AD 1020, made the following remarks in his *Kitab Tahdid*.

> ‘An eclipse does not become clear to a beholder until the portion formed of it, according to some authors of *zijes*, amounts to one digit. I mean a part of one twelfth of its size.... I consider the amount of a digit in this connection to be excessive because a small immersion can be seen...’ (trans. Ali 1967).

A phase of one-twelfth would be attained about 5 min after true first contact for a central eclipse. However, al-Biruni is implying a smaller delay in detection than this.

3. Even, if there were appreciable bias in discerning the start or end of an eclipse, the fact that both the Arab and Chinese observations are about equally divided between first and last contacts implies that the bias in the net result should be small because the effect is equal and opposite at first and last contacts.

In our opinion, the constraints imposed by the untimed solar eclipses, taken together with the timed data discussed above, provide strong evidence in favour of the spline curve over the parabola. Hence, we assert that the non-tidal acceleration of the Earth's rotation has not been constant over the past 2700 years. However, it might well be the composite of a smooth trend and a variable component which have different geophysical origins. We return to this point in § 5 *d*.

(*c*) *Accuracy of the timed data*

The distributions of the residuals of the Chinese and Arab timings plotted in figure 6 permit us to estimate the errors in timing. The standard deviations are 13 and 9 min for the Chinese and Arab observations, respectively. As reported in § 3 *a* (ii), the Chinese astronomers timed most of their observations of eclipses to the nearest *ke*, which is $\frac{1}{100}$ of a day (= 14 min). The maximum range in error, therefore, should be ±7 min, if the dominant source of error was the imprecision of the unit of measurement. The computed standard deviation of 13 min leads us to conclude, however, that the uncertainty of the observations is about one *ke* (14 min).

The Arab astronomers (see § 3 *a* (iv)) timed eclipses by measuring the altitude of the Sun, the Moon, or a bright star to the nearest degree, which corresponds to 4 min. Our estimate for the standard deviation is 9 min. For the subset of Arab timings of solar eclipses (shaded in figure 6*a*), the standard deviation is 6 min, which implies that the greatest errors were made in timing the lunar eclipses (at night). The imprecision of the unit of measurement was not the limiting factor of the timing accuracy. Their best accuracy was somewhat greater than one unit (4 min).

From the frequency distribution of the residuals of all the Babylonian timings of eclipses, we find an estimate for the standard deviation of about $\frac{1}{2}$ h. All but the very earliest lunar eclipses were recorded to the nearest degree (4 min) (see § 3 *a* (i)). Thus again, the unit of measurement was not the limiting factor in the timing, nor was it likely to have been the nature of the phenomena (sunset/sunrise and the beginning/end of a lunar eclipse). it is more likely to have been the variable rate of their clocks which were probably some kind of clepsydra. Supporting evidence for this comes from the fact that the standard deviation for the restricted set of lunar eclipses which ocurred within an hour of sunset/sunrise (black dots in figure 5), is found to be 13 min, after the rejection of one outlier.

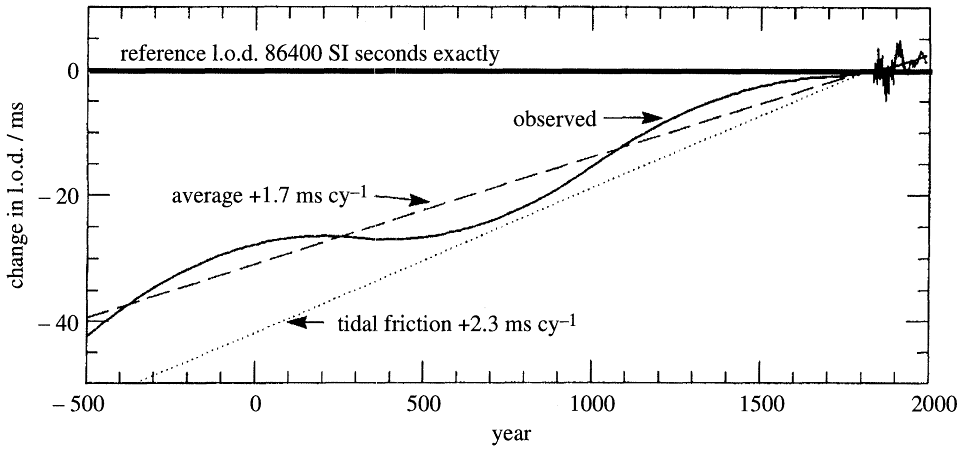

Figure 7. Changes in the l.o.d. −500 to +1990 obtained by taking the first time derivative along the spline curve shown in figures 2–5. The high-frequency changes in the l.o.d. +1830 to +1990 are taken from Jordi *et al.* (1994).

(*d*) *Change in l.o.d.*

By taking the first derivative along the cubic spline in figures 2–5, we derive the change in the l.o.d. This is plotted in figure 7, together with the fluctuations in the l.o.d. from 1830 to the present, taken from Jordi *et al.* (1994). This shows an average observed trend of $+1.7\,\mathrm{ms\,cy}^{-1}$ compared with $+2.3 \pm 0.1\,\mathrm{ms\,cy}^{-1}$, predicted from tidal friction. There is thus an accelerative component producing a rate of shortening of the l.o.d. of $-0.6 \pm 0.1\,\mathrm{ms\,cy}^{-1}$ and, since there is no evidence to suggest that the tidal dissipation has altered significantly over the past 2700 years, we conclude that the observed long-term fluctuation revealed in figure 7 is not tidal in origin. It has a semi-amplitude of ~ 4 ms about the average trend of $-1.7\,\mathrm{ms\,cy}^{-1}$, and is thus comparable in amplitude to the decade fluctuations, but with a much longer period of ~ 1500 yr.

As mentioned above, the non-tidal acceleration and fluctuation may have different geophysical origins. The non-tidal acceleration may be associated with the rate of change in the Earth's oblateness, attributed to viscous rebound of the solid Earth from the decrease in load following the last deglaciation. From an analysis of the acceleration of the node of the orbit of the Lageos satellite, Yoder *et al.* (1983) deduced a present-day rate of change of the Earth's zonal harmonic J_2 of $-3 \times 10^{-11}\,\mathrm{yr}^{-1}$ which implies an acceleration in the Earth's rotation equivalent to a rate of change in the l.o.d. of $-0.6\,\mathrm{ms\,cy}^{-1}$. A more recent result by Cheng *et al.* (1989) from Starlette data gives $(-2.5 \pm 0.3) \times 10^{-11}\,\mathrm{yr}^{-1}$, which implies a rate of $-0.44 \pm 0.05\,\mathrm{ms\,cy}^{-1}$ in the l.o.d.

Our result of $-0.6 \pm 0.1\,\mathrm{ms\,cy}^{-1}$ is an average over the past 2500 years, and taken together with the Starlette result implies that the rate of change of J_2 in the past was greater than it is now. This is expected from the theory of post-glacial rebound. Assuming an exponential decay of $\dot{J}_2$ between time in the past, t, and time now, t_0, we have

$$\dot{J}_2(t) = \dot{J}_2(t_0)\mathrm{e}^{(t_0-t)/\tau},$$

where τ is the relaxation time of about 4000 yr. Correspondingly, the rate of

Table 2. *Values of* ΔT *and l.o.d. taken from the spline curve shown in figures 2–5.*

year	ΔT (s)	l.o.d. (ms)	year	ΔT (s)	l.o.d. (ms)	year	ΔT (s)	l.o.d. (ms)	year	ΔT (s)	l.o.d. (ms)
−500	16 800	−42	0	10 600	−28	500	5700	−27	1000	1600	−15
−450	16 000	−40	50	10 100	−27	550	5200	−26	1050	1350	−14
−400	15 300	−38	100	9600	−27	600	4700	−26	1100	1100	−12
−350	14 600	−36	150	9100	−27	650	4300	−25	1150	900	−10
−300	14 000	−35	200	8600	−26	700	3800	−24	1200	750	−9
−250	13 400	−33	250	8200	−27	750	3400	−23	1250	600	−7
−200	12 800	−32	300	7700	−27	800	3000	−22	1300	460	−6
−150	12 200	−30	350	7200	−27	850	2600	−20	1350	360	−5
−100	11 600	−29	400	6700	−27	900	2200	−19	1400	280	−4
−50	11 100	−29	450	6200	−27	950	1900	−17	1450	200	−3
									1500	150	−3
									1550	110	−2
									1600	80	−2

change in the l.o.d., $\dot{\mathrm{d}}$, is given by

$$\dot{\mathrm{d}}(t) = \dot{\mathrm{d}}(t_0)\mathrm{e}^{(t_0-t)/\tau}.$$

Adopting $\dot{\mathrm{d}}(t_0) = 0.44\,\mathrm{ms\,cy^{-1}}$ and $\tau = 40$ cy, and integrating between t_0 and t, we find the shortening of the l.o.d. at epoch t due to post-glacial rebound to be

$$\mathrm{d}(t) = +40 \times 0.44 \times (\mathrm{e}^{(18.2-t)/40} - 1).$$

At the epoch −500 ($t = -5$), this has the value +14 ms, and, added to the expected tidal value of −53 ms, gives a resultant of −39 ms, which is close to the value of the dashed line drawn in figure 7. Indeed, to within the accuracy of the data, the expression for d(t) above, added algebraically to the tidal value, is not significantly different from the trend of $+1.70\,\mathrm{ms\,cy^{-1}}$ plotted in figure 7. So our result for the non-tidal 'acceleration' over the past 2500 years is consistent with modern satellite measurements of $\dot{J}_2$ combined with a relaxation time of 4000 yr for post-glacial rebound. However, at epoch −500, the value of d(t) converges to +10 ms for large relaxation times, giving a resultant limit of −43 ms in figure 7, which is still within the bounds of possibility given the obfuscating effect of the long-term fluctuation. Therefore, our data do not set an upper bound on the relaxation time. A longer time base would help to tighten this constraint, and reliable eclipse records from circa 1000 BC or earlier would be useful. Unfortunately, we are not aware of any at present.

The quasi-periodic fluctuation on a timescale of about 1500 yr might well arise from core–mantle coupling, which is the mechanism most favoured for the decade fluctuations (Lambeck 1980). The maximum angular acceleration is $\sim \pm 2\,\mathrm{ms\,cy^{-1}}$, which corresponds to $\sim \pm 5 \times 10^{-22}\,\mathrm{rad\,s^{-2}}$. The magnitude of the torque operating on the mantle is thus $\sim \pm 4 \times 10^{16}$ Nm.

The decade fluctuations revealed after the introduction of the telescope (see figure 7) are no doubt present throughout the entire period of this investigation, but the integral of these fluctuations is too small to be detected in the pre-telescopic results for ΔT.

6. Conclusions

The following conclusions can be drawn from this analysis of all the reliable historical observations in the period 700 BC to AD 1990 which are pertinent to the question of the long-term variability of the Earth's rate of rotation.

1. The average observed increase in the l.o.d. over the past 2700 years is $(+1.70 \pm 0.05)$ ms cy^{-1}, which is equivalent to an acceleration in spin of $(-4.5 \pm 0.1) \times 10^{-22}$ rad s^{-2}.

2. This average observed value is the composite of two actual trends which arise from different physical causes.

The first cause, which is due to tidal dissipation, produces an increase in the l.o.d. of $(+2.3 \pm 0.1)$ ms cy^{-1} $(\equiv (-6.1 \pm 0.4) \times 10^{-22}$ rad s$^{-2})$. This value for the tidal braking of the Earth's spin is commensurate with a tidal acceleration of the Moon of $-26.0''$ cy^{-2}.

The second cause, which is non-tidal in origin, produces an average rate of shortening of the l.o.d. of (-0.6 ± 0.1) ms cy^{-1} $(\equiv (+1.6 \pm 0.4) \times 10^{-22}$ rad s$^{-2})$. This result is consistent with modern values of $\dot{J}_2$ and the theory of post-glacial rebound.

3. There is a long-term fluctuation in the l.o.d., with a semi-amplitude of ~ 4 ms and a period of ~ 1500 yr. This may have its origin in core–mantle coupling.

4. A good approximation to the value of ΔT over the past 2700 years, with an error not exceeding 8 min, is given by the parabola $+31t^2$ s, where t is measured in centuries from the epoch AD 1820. For precise predictions of the circumstances of eclipses before AD 1600, the values listed in table 2, which are evaluated from the spline curve, should be used. These values are based on a lunar tidal acceleration of $-26.0''$ cy^{-2}. A correction of $+0.5''$ cy^{-2} to this value produces a change of $-0.46t^2$ s in ΔT.

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